HEADCUT DEVELOPMENT IN VEGETATED EARTH SPILLWAYS

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ABSTRACT. Vegetated earth (soil and rock) auxiliary or emergency spillways have been used extensively on *flood* control reservoirs within the United States. Despite their widespread use, the processes by which these spillways erode during extreme events are only imperfectly understood, and there is a need for procedure to better predict spillway performance and safety. Therefore, research utilizing both laboratory and field data was undertaken to improve criteria for design and analysis of these spillways. For computational purposes, it was found that erosion of vegetated earth spillways could be divided into three phases. These phases are vegetal cover failure, conentrated flow erosion, and headcut advance. A computational procedure is developed for predicting the time associated with the first two of these phases. The procedure combines simplified flow and detachment rate relations in a form intended to minimize data requirements while allowing application to a broad range of conditions. Results of applying the procedure to predict headcut formation are shown to be generally consistent with availabl field data. This procedure may, therefore, be used to estimate the time of headcut formation for given flow and channel surface conditions. Keywords. Dam, Spillway, Headcut, Erosion, Vegetated channels.

egetated earth (soil and rock) auxiliary or emergency spillways have been used extensively on flood control reservoirs within the United States, These spillways generally consist of a trapezoidal channel cut through natural materials and topsoiled and vegetated as appropriate for the local area. Because these channels are an attractive alternative to structural spillways for both economic and aesthetic reasons, the USDA-Soil Conservation Service (SCS) has constructed approximately 23,000 structures utilizing this type of spillway (Cato and Mathewson, 1989).

Despite their widespread successful use, the processes by which these spillways resist failure are complex and only imperfectly understood. The development of design principles has occurred primarily during the last 40 years (Ralston and Brevard, 1988), and the present design criteria (USDA-SCS, 1973, 1985) were developed from limited data. Therefore, the SCS and Agricultural Research Service (ARS) have developed an ongoing program with the goal of increasing understanding of the processes and improving the criteria for design and analysis.

In 1983, the SCS formed the Emergency Spillway Flow Study Task Group (ESFSTG) to gather data from spillways which had experienced greater than 0.9 m (3 ft) of head or sustained major damage. That effort, which is still continuing, resulted in the acquisition and compilation of data from 83 sites representing 13 flood events in 10 states. Data from three additional flood events are in the process of being gathered and compiled. The ARS has cooperated in the data acquisition effort since the formation of the ESFSTG as well as conducting laboratory research to quantify the associated erosion processes.

The joint **ARS/SCS** Design and Analysis of Earth Spillways team was formed in 1991 for the purpose of developing and documenting new technology for use in the design and analysis of earth spillways. The goal of this team has been to utilize the laboratory and field data and experience as the basis for development of as straightforward and robust a computational algorithm as possible while minimizing data input requirements. The following analysis represents a portion of that effort carried out by the authors in consultation with other team members.

EROSION PROCESSES

Erosional damage to vegetated earth spillways is normally initiated in regions of supercritical flow and is observed to be a three-phase process. The first phase is erosion resulting in the local failure and removal of the vegetal cover, if any, and the development of concentrated flow. The second phase is the downward and downstream erosion resulting from the flow and stress concentration leading to the formation of a vertical or near vertical headcut. The third phase of the erosion process is the upstream advance of the headcut, which may also be accompanied by further widening and deepening. Only the first two phases will be discussed herein. Work is continuing on quantification of the headcut advance phase.

PHASE 1

Phase 1 erosion results in the local failure and removal of the vegetal cover to form an area of flow concentration. The basis for estimating the time of the vegetal cover failure has been discussed previously (Temple, 1992) and

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will be reviewed only briefly here. If it is assumed that the failure is the result of erosion through the vegetal cover as observed in the laboratory, the governing relations may be written as:

$$\dot{\varepsilon} = k_d (\tau_e - \tau_c) \tag{1}$$

with

$$\tau_{e} = \gamma \, \mathrm{dS} \left(1 - \dot{C}_{F} \right) \left(\frac{n_{s}}{n} \right)^{2} \tag{2}$$

where

- ϵ = erosion or detachment rate in volume per unit area per unit time
- \mathbf{k}_{d} = detachment rate coefficient in volume per unit area per unit time per unit stress
- τ_e = the erosionally effective stress on the soil
- τ_c = threshold or critical soil stress y = unit weight of water
- d = flow depth
- S = slope of the energy grade line
- C_{F} = vegetal cover factor
- $n_s = soil grain roughness expressed in terms of$ Manning's coefficient
- $\mathbf{n} = \text{Manning's } \mathbf{n}$

If steady uniform flow is assumed for application, d becomes the normal flow depth and S the channel bed slope. Design aids for the determination of C_F , n_s , and n for vegetated conditions are provided by Temple et al. (1987).

Representing the topsoil properties by the plasticity index, integrating equation 1 to the point of vegetal failure under the assumption that $\tau_e >> \tau_c$, and calibrating with data from field spillways, yields a relation describing time of vegetal cover failure as (Temple, 1992):

$$\int_{0}^{t} \tau_{e} dt = 9 I_{w} + 50$$
 (3)

where t_f is the time of vegetal cover failure and I, is the plasticity index of the soil with τ_{e} in Pascals and t in hours. For τ_{e} in lb/ft², equation 3 becomes:

$$\int_{0}^{t_{\rm f}} \tau_{\rm e} {\rm d}t = 0.19 \, {\rm I}_{\rm w} + 1.0 \tag{3a}$$

With the spillway hydrograph known, t_f of equation 3 may be determined using numerical integration. Integration of the entire hydrograph with the left side of equation 3 less than the right would indicate insufficient attack to generate failure.

As indicated, equation 3 is applicable only when the mode of vegetal cover failure is erosion through the vegetal cover. When the vegetation is rooted in a shallow topsoil over a material which the roots cannot penetrate, the observed mode of failure is a stripping or rafting of the sod off of the lower material. As shown by the square symbols

in figure 1, this stripping action was observed to occur in field spillways at comparatively low stresses when the average potential rooting depth was less than approximately 0.3 m (1 ft). Since the stripping action tends to propagate downstream from a local weak area, it may be speculated that less rooting depth would be required if the depth was uniform over the entire spillway surface.

Although not observed in the laboratory, it may also be speculated that for very large stresses, the mode of failure may shift from erosion through the vegetation to general destruction of the vegetal cover by the gross turbulent hydraulic stress. It therefore seems prudent to limit the range of application of equation 3 by limiting the maximum total hydraulic stress. The line in figure 1 may be used to accomplish this purpose as well as to account for the sod stripping action experienced by shallow rooted covers. Stresses to the right of the line are assumed to result in instantaneous cover failure. Because gross stress generated instantaneous failure has not been observed directly, the position of the line on the right edge of the figure is somewhat subjective and may require modification as more data becomes available. In its present form, it serves simply to bound application of equation 3. In metric units, this bounding line is described by the relation:

$$\frac{D_r}{0.46} = 2^8 \left(\frac{\tau}{646} - \frac{1}{2}\right)^9 + \frac{1}{2}$$
(4)

with D_r as the available rooting depth in meters and τ as the the gross hydraulic stress in Pa.

For D_r in ft and τ in lb/ft^2 :

$$\frac{D_{\rm r}}{1.5} = 2^8 \left(\frac{\tau}{13.5} - \frac{1}{2}\right)^9 + \frac{1}{2}$$
(4a)

Although equation 2 was initially developed for application to uniform vegetal covers, it may also be used to estimate the erosionally effective stress within typical spillway cover discontinuities in supercritical flow without the use of three-dimensional or spatially varied flow



Figure I-Peak stress vs. potential rooting depth of the cover.

models. The vegetal cover factor, CF, represents local conditions (Temple et al., 1987). Therefore, minor discontinuities in the cover, such as roads or trails across the spillway, which have a maximum dimension parallel to the flow on the order of flow depth, may be accounted for by using the cover factor associated with conditions within the discontinuity. If the vegetation is entirely removed, the appropriate value of the cover factor is zero.

For major discontinuities in the cover, such as roads or trails parallel to the flow, the value of n in equation 2 must be adjusted to reflect local conditions as well. For a trail or roadway with all vegetal cover removed, n would typically tend to a value of approximately 0.02. However, the value of n used in computing the flow depth in equation 2 is that appropriate for the spillway in general. This approach assumes the width of the discontinuity to be large enough that shear across vertical planes parallel to the principal flow direction may be ignored, narrow enough that flow concentration does not meaningfully influence discharge outside of the discontinuity, and long enough for full flow concentration to develop. This is equivalent to computing bed stress based on a wide channel with variable discharge, energy slope equal to bed slope, and flow depth equal to the flow depth of the overall channel. The effect of the vegetation in this case is to concentrate the flow within the discontinuity. This effect also exists in phase 2 of the erosion process.

PHASE 2

Phase 2 of the vegetated earth spillway erosion process is concentrated flow erosion representing a transition from phase 1 to phase 3 (headcut advance). It begins when all vegetal protection has been removed and a flow concentration formed, and ends when the erosion is sufficient to cause plunging action with associated energy and stress concentrations at the base of the headcut. It is similar to the major discontinuity described above, except that all vegetal protection has been locally removed, and different materials may be encountered as erosion depth is increased.

If exact dimensions of the eroding area were known, three-dimensional or spatially varied flow computations could be used to determine details of flow behavior. However, the entire process is discontinuity driven, and dimensions often depend on cover and geologic material variations that cannot be reasonably predicted. Therefore, the same simplifying length and width assumptions are made for the flow concentration area in phase 2 erosion as were made for cover discontinuities in phase 1. If it is further assumed that the bed slope within the area of concentrated flow is equal to the channel bed slope, erosion is detachment limited, and that all roughness elements in the flow concentration area may potentially be detached by the flow, then the erosionally effective stress is equal to the gross stress and may be computed by:

$$\tau_e = \tau = \gamma (d+h)S \tag{5}$$

where

- d = flow depth outside of the concentrated flow area
- h = eroded depth in the area of the flow and stress concentration

Equation 5 is a special case of equation 2 under the conditions of phase 2 erosion.

Since it is desirable to perform computations for a broad range of materials, and equation 5 requires that erosion depth be tracked, it is necessary in this phase to be able to determine τ_c and k_d of equation 1 directly. Although a number of researchers have assumed this form of a detachment rate relation, a review of the literature does not reveal any general agreement on determining the value of the parameters for various materials.

By definition, critical stress, τ_c , is the time averaged stress at which particle motion begins. Although conceptually simple, determination of this stress is not always straightforward. For coarse material, determination of essential parameters such as flow depth may be difficult. For fine material, experimental determination of the point of incipient motion requires either the application of subjective judgment or assumptions related to flow boundary interaction at low sediment transport rates. Form roughness further complicates erosionally effective stress determination. Researchers, including Lavelle and Mofjeld (1987) have questioned whether incipient motion actually exists. The concept has, however, been found functionally useful despite its shortcomings.

For material which exists as discrete particles, Shields' diagram is generally used as the basis for determining critical stress for incipient motion (ASCE. 1966). This approach is adopted for the analysis herein with the representative diameter taken as the cube root of the particle volume. For larger *in situ* geologic materials, this approach becomes more of an approximation because particle shape and interlocking are not accounted for. For the near surface conditions of interest in hcadcut formation, this will generally be less of a problem than for deeper materials because of the tendency for the materials near the surface to have been previously disturbed. Additional study may allow prediction of critical stress for previously undisturbed geologic materials to be refined in the future.

If it is truly the point of incipient motion that is sought, it may be argued that the same criterion should apply to both cohesive and noncohesive materials, since there will generally be some "free" particles at the surface under submerged conditions. Using this logic, the interparticle forces influence the rate of detachment rather than the threshold. As may be seen from figure 2, this is essentially equivalent to the assumption of negligibly small critical shear ($\tau_c \approx 0$) for fine grained materials made in the phase 1 discussion.

To determine an appropriate soil detachment rate coefficient for use in the present application, the available literature was searched for open channel erosion rate data. Ten studies covering 98 soil materials were identified for which erosion rate and tractive stress data could be extracted. These materials were all tine grained in the sense of the discussion of the preceding paragraph. The data sources were Partheniades (1965). McWhorter et al. (1968). Arulanandan et al. (1980), Fuduka and Lick (1980), Chen and Anderson (1987), Shaikh et al. (1987), Clopper and Chen (1988), Elliot et al. (1989), Grissinger et al. (1989). and Hanson (1990). with 60% of the data coming from Arulanandan et al. (1980) and Elliot et al. (1989). The range of erosion rates and stresses represented is shown in figure 3.



Figure 2–Critical stress for incipient motion computed from Shields' criteria with a sediment specific gravity of 2.65 and a kinematic viscosity of water of $9.3 \times 10^{-7} m^2/s (10.5 ft^2/s)$.

The material parameters common between the studies were percent clay, dry density, and plasticity index. Regression analysis of simple equation forms was used to produce the relation in metric units:

$$k_{d} = \frac{10\gamma_{w}}{\gamma_{d}} \exp\left(-0.121c_{\%}^{0.406} \frac{\gamma_{d}}{_{0}\gamma_{w}}^{3.10}\right)$$
(6)

where

 k_d = erosion rate in units of [(cm³/N-s)]

 $\mathbf{c}_{\mathbf{g}_{n}} = \text{percent clay}$

- γ_d = dry unit weight in Mg/m³
- $\gamma_{\rm w}$ = unit weight of water in Mg/m³

The relation in English units is:

$$k_{d} = \frac{5.66 \gamma_{w}}{\gamma_{d}} \exp\left(-0.121 c_{\%}^{0.406} \left(\frac{\gamma_{d}}{\gamma_{w}}\right)^{3.10}\right) \quad (6a)$$

where



Figure 3-Range of erosion rate data by source.

- k_d = erosion rate in units of [(ft³/lb-h)]
- γ_d = dry unit weight in lb/ft³
- $\gamma_{\rm w}$ = unit weight of water in lb/ft³

The value of the coefficient of multiple determination, r^2 , for equation 6 was 0.71. Inclusion of plasticity index was not found to improve the relation for the data evaluated. Data suitable for refining the relation for larger noncohesive material were not identified.

Equations 1, 5, and 6 along with figure 2 provide a basis for tracking erosion depth under the stated assumptions. Use of figure 2 and equation 6 for the entire range of materials results in the erosion rate coefficient being the key parameter governing the performance of cohesive materials, and critical stress being the key parameter governing the performance of noncohesive materials. Qualitatively, this is consistent with observed behavior.

The actual depth of erosion at the end of phase 1 (vegetal cover removal) and beginning of phase 2 will depend on a number of local factors related to the properties of the cover, the flow, and the surface materials. It is not presently possible to predict this value for a specific set of conditions with any degree of confidence. Therefore, a value of 0.15 m (0.5 ft) or the available rooting depth, whichever is less, is assumed for h at the beginning of phase 2 for purposes of general application to the problem of heacut prediction in spillways. This value is selected based on subjective evaluation of conditions observed in the field and laboratory. There are presently insufficient quantitative data to make an objective determination of the appropriate value(s) to use.

The end of phase 2 (concentrated flow surface detachment) and the beginning of phase 3 erosion (headcut advance) is defined as the point where the flow plunges and develops a vertical face with increased stress and flow energy dissipation in the area of the base. It is, therefore, discharge dependent. Although actual conditions at transition will also depend on slope and downstream conditions, a first order estimate may be obtained by examining submergence of an over-fall onto a horizontal floor with a free exit, and critical conditions at the overfall brink (fig. 4). Under these conditions, a vented nappe will tend to be supported $(d_f = h)$ whenever the overfall height, h, is less than or equal to the flow critical depth, d_c (Rand, 1955). For h greater than d_c , the nappe will be unsupported and will tend to plunge downward. Therefore, for applications where more detailed information is not available, the end of phase 2 and beginning of phase 3 erosion may be approximated by the condition of erosion depth equal to critical depth. For erosion depth greater than critical depth, the potential would exist for the headcut to advance upstream.

APPLICATION TO FIELD **DATA**

The intended application of the equations of the preceding section is the prediction of the time required to develop a headcut in a specific spillway reach subjected to a known flood hydrograph. Field data suitable for direct evaluation of these relations for that purpose are difficult to obtain. Since spillways are generally designed for only a 1 to 4% chance of operation in any given year, it is not possible to determine which spillways will flow, and to acquire data in advance of a flood event. During the flood,



Figure 4-Defining sketch for overfall conditions.

the spillways are often inaccessible because of local flooding and storm conditions. Therefore, it is only after the event that data may be obtained. This means that the quality of the data gathered by the ESFSTG related to preflood conditions is variable, and the time of headcut formation is not directly available. The quality of data describing the earth materials is similarly variable with laboratory testing performed on materials in some instances, and only visual analysis of what had apparently been eroded available in others.

At least a qualitative description of the apparent initial surface cover conditions and earth materials was entered for each of the spillway reaches entered in the compiled data base. When additional information was available from direct observation or testing, that information was also entered. All of the information needed to apply equations I through 6 was directly available for only a few entries. Therefore, the default values given in tables 1 and 2 were established so that applicability could be evaluated in general terms for the entire compiled data base. The default values were based on experience and qualitative comparison of the measured and unmeasured conditions.

Spillway hydrographs were simulated from available rainfall and reservoir high water data by SCS hydrologists using DAMS2 software. Normal flow depth was assumed in all applications of equations 2 and 5. Required mathematical integrations were performed using the same time increment as had been used in the initial hydrograph generation.

Although it was not possible to directly determine headcut formation time for specific reaches of field spillways, it was possible to compare predicted and observed performance in more general terms by defining headcut formation in terms of the critical depth associated with maximum discharge. By using the default values in tables 1 and 2 to fill data gaps, the data base contained 109 spillway reach entries suitable for evaluation. These included 58 entries which had been used in the development of equation 3, and 11 entries which had been

Table 1. Default vegetal parameters used when directly determined values are unavailable

	Exit Cha	nnel	Natural Hillslope		
Cover Description	Retardance	Cover	Retardance	Cover	
	Curve Index	Factor	Curve Index	Factor	
Good Vegetal Cover	5.6	0.75	5.1	0.55	
Fair Vegetal Cover	5.0	0.50	4.5	0.30	
Poor Vegetal Cover	4.4	0.25	3.9	0.05	

 Table 2. Default material parameters used when measured values are unavailable

Material Description*	Plasticity	Bulk Dry Density		Percent	Representativ Diameter	
	Index	Mg/m ³	lb/ft ³	Clay	mm	in.
Class I Rock	0	1.6	100	0	1100	43
Class 11 Rock	0	1.6	100	0	610	24
Class 111 Rock	0	1.6	100	0	150	6
Erosion Resista	nt					
Soil	20	1.6	100	30	0.2	0.008
Erodible Soil	10	1.6	100	20	0.2	0.008
Easily Eroded	Soil 0	1.6	100	0	0.2	0.008

 Visual classification. Rock class follows that defined by USDA-SCS (1987).

used in the development of equation 4. Entries not previously used included both those with missing cover data, and those entered into the data base after the initial analysis was performed.

Predicted and observed conditions were consistent for 8 1 of the 109 entries with 44 of these having developed headcuts, 12 having experienced sod stripping action with negligible erosion of the underlying material, and 25 having experienced negligible damage. In terms of the identified erosion phases, the 44 with headcuts had entered phase 3, the 12 sod stripped reaches were still in phase 2, and the 25 with negligible damage were still in phase 1 at the end of the hydrograph.

For 21 of the 109 entries evaluated, more damage was predicted than observed. For 6 of the 21, headcuts were predicted where negligible damage was observed, indicating conservative prediction of phase 1 failure. For one site, sod stripping was predicted where negligible damage was observed. For 10 of the 21, headcuts were predicted where only minor surface erosion was observed, and for 4 reaches, headcuts were predicted where only sod stripping was observed. Reach by reach examination of these latter 14 reaches suggested overestimation of the detachment rate coefficient, $\mathbf{k}_{\mathbf{d}}$, (underestimation of material erosion resistance) to be the most common probable cause of the inconsistency. The fact that equation 6 does not account for interparticle bonding except through percent clay and density appeared to be partially responsible. In future application, this may be overcome for soil materials by direct measurement of erodibility as described by Hanson (1991).

For 7 of the 109 reaches, less damage was predicted than observed. For five of these, negligible damage was predicted and minor surface erosion observed. Errors in estimating initial cover conditions are the most likely cause in all five cases. For one reach, sod stripping with no subsequent erosion was predicted, and a shallow headcut was observed. Properties of the eroded material were questionable in this case. For one reach, negligible damage was predicted, and a headcut was observed to have formed. This reach was constructed through easily eroded material and entered in the data base as having an excellent cover of bermuda grass prior to flow. Re-examination of the data exposed the probability of bank seepage and sloughing in the area where the headcut initially formed. If this type of initial discontinuity is assumed, headcut formation consistent with observation is predicted.

SUMMARY AND CONCLUSIONS

For computational purposes, erosion of vegetated earth spillways may be divided into the phases of vegetal failure, concentrated flow erosion, and headcut advance. A computational procedure was developed for predicting the time associated with the first two of these phases. The procedure combines simplified flow and detachment rate relations in a form intended to minimize data requirements while allowing application to a broad range of conditions. Erosion was assumed to be detachment limited in all cases, with Shields' criteria describing the detachment threshold. A simplified relation for predicting an erosion rate coefficient was developed from published data.

All relations were developed based on conditions in a unit width of the spillway. Cover discontinuity and developing headcut width conditions yielding the maximum bottom stress were assumed. This approach was taken to simplify computations and allow two-dimensional geologic information to be used. Supercritical flow conditions capable of expanding the area of the developing headcut downstream were also assumed.

The utility of the relations and the computational format were illustrated by application to the available data from field spillways. The nature of the available data did not, however, allow direct comparison of time of headcut formation. It is also noted that a subset of the data had previously been used in developing the phase 1 relations. Within the noted constraints and limitations, the results of application of the procedure to field data were generally consistent with observed performance.

The goal of the present effort was to develop relations for use in a computational algorithm which was as easily understood, robust, and applicable to as broad a range of conditions as possible. To satisfy this goal, approximations and simplifications were required as noted in the previous sections. Therefore, computational refinements resulting from future research and analysis should be expected and encouraged. The presented procedure may be used to estimate the time to headcut formation until more refined procedures are developed.

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