

CHAPTER E-2 REINFORCED CONCRETE FAILURE MECHANISMS

E-2.1 Key Concepts and Factors Affecting Risks

This section discusses the failure mechanisms of reinforced concrete members such as piers, walls, slabs, and buttresses associated with a variety of hydraulic structures including concrete dams, spillways, outlet works, floodwalls and locks. This section includes the following key concepts: 1) factors influencing the strength and stability of the reinforced concrete sections, 2) considerations when assigning failure probabilities based on structural analysis results, 3) considerations related to National codes (such as ACI or AASHTO) in the risk context, and 4) a typical event tree of the failure progression.

E-2.1.1 Factors Influencing Strength and Stability

Factors influencing the stability of reinforced sections include:

- geometry and support conditions of the section,
- material properties of the reinforcement,
- material properties of the concrete,
- amount and detailing of the reinforcement,
- type and duration of loading, and
- location of the reinforced concrete members within the structural system

E-2.1.2 Geometry and Support Conditions

As illustrated in Figure E-2-1, reinforced concrete sections in hydraulic structures vary greatly in size and shape. Spillway walls and floodwalls can be very tall and narrow as shown on Figures E-2-2 and E-2-3. Spillway piers tend to be shorter and wider than walls as shown on Figure E-2-4. Buttresses in slab and buttress dams can vary from very thin tall sections like at Stony Gorge Dam to more stout sections like at Coolidge Dam as shown on Figure E-2-5. The geometry of the concrete section has a large impact on how the section may fail. As a rule of thumb, sections with height to thickness ratios of 4:1 or less tend to slide more than rotate or bend while sections

with height to width ratios more than 4:1 tend to bend, rotate and topple (4:1 ratio based on deep beam criteria in ACI Code 318).

Structures have definite, signature dynamic characteristics. The geometry greatly affects the natural frequency of the reinforced concrete member and how the frequency content of an earthquake ground motion matches up with the natural frequency of the member. The natural frequency of the member decreases as the height to width ratio increases. In other words, the section becomes more flexible as the section becomes taller and thinner as shown on Figure E-2-6.

Figures E-2-7 through E-2-9 provide useful equations to compute the natural frequencies of various concrete structures. As a check, the natural frequency of the member should typically be compared to the acceleration response spectra at 5 percent damping to judge how seismic loads may amplify through the section. For example, a concrete member with a natural frequency of 3 Hz on a rock site might have significant amplification of the seismic load from the base to the top of the section. If the structure cracks, a reduced modulus (typically 1/3 the uncracked modulus) can be used to get a sense as to how the amplification might change.

Often times if a site specific probabilistic seismic hazard analysis (PSHA) is performed, a uniform hazard spectra similar to the one shown on Figure E-2-10 is developed that can be conservatively used for the purpose of estimating structural acceleration response based on a computed or estimated structural fundamental frequency.

Foundation support conditions can greatly influence the dynamic response of a structure. The dynamic responses of a structure founded on rock will be different than the dynamic response of the same structure to the same earthquake if it is founded on soil. Design criteria in structural design codes reflect this difference in the form of site classifications. Structures founded on the top of dams must be evaluated considering amplification of earthquake ground motions through the dam.

E-2.1.3 Reinforcement Material Properties

Material properties of the reinforcing steel directly contribute to the strength of the concrete section. The modulus of elasticity of steel is fairly consistent at 29,000,000 lb/in². However, the yield strength and ultimate strength of the steel depends on the era of the structure as shown in Table E-2-1. The yield strength and ultimate strength by historical ASTM designations and federal specification designations is also indicated in Table E-2-1.

The density of steel usually does not impact the stability of the structure because the weight of the reinforcing bars is significantly less than the weight of the concrete section. The shear strength of the reinforcement is typically taken as the tensile yield strength. However, when loaded in combined tension and shear, the strengths may not be additive.

Table E-2-1 Reinforcing Bars 1911 to Present; ASTM Specifications; Minimum Yield and Tensile Strengths in psi (Adapted from ASCE 41 Seismic Rehabilitation of Existing Buildings)

ASTM or Federal Spec	Year Range		Steel Type	Grade 33 (Structural)		Grade 40 (Intermediate)		Grade 50 (Hard)		Grade 60		Grade 75	
	Start	End		Min. Yield	Min. Tensile	Min. Yield	Min. Tensile	Min. Yield	Min. Tensile	Min. Yield	Min. Tensile	Min. Yield	Min. Tensile
A15	1911	1966	Billet	33,000	55,000	40,000	70,000	50,000	80,000				
A408	1957	1966	Billet	33,000	55,000	40,000	70,000	50,000	80,000				
A432	1959	1966	Billet							60,000	90,000		
A431	1959	1966	Billet									75,000	100,000
A615	1968	1972	Billet			40,000	70,000			60,000	90,000	75,000	100,000
A615	1974	1986	Billet			40,000	70,000			60,000	90,000		
A615	1987	Present	Billet			40,000	70,000			60,000	90,000	75,000	100,000
A16	1913	1966	Rail					50,000	80,000				
A61	1963	1966	Rail							60,000	90,000		
A616	1968	1999	Rail					50,000	80,000	60,000	90,000		
A160	1936	1964	Axle	33,000	55,000	40,000	70,000	50,000	80,000				
A160	1965	1966	Axle	33,000	55,000	40,000	70,000	50,000	80,000	60,000	90,000		
A617	1968	1999	Axle			40,000	70,000			60,000	90,000		
A996	2000	Present	Rail, Axle			40,000	70,000	50,000	80,000	60,000	90,000		
A706	1974	Present	Low-Alloy							60,000	80,000		
A955M	1996	Present	Stainless			40,000	70,000			60,000	90,000	75,000	100,000
QQ-B-71a	1938	1957	Billet	33,000	55,000	40,000	70,000	50,000	80,000				
QQ-S-632	1957	1963	Billet			40,000	70,000			60,000	90,000	75,000	100,000

E-2.1.4 Concrete Material Properties

Material properties of the concrete also directly contribute to the strength of the reinforced concrete member and vary from one structure to the next. Concrete material properties needed to estimate the strength of a reinforced concrete member are the density, modulus of elasticity, compressive strength, tensile strength, and shear strength. If laboratory data is not available, 28-day design compressive strength values can often be obtained from the design drawings or original project specifications. Many times, the 28-day design compressive strength value obtained from design documents is then used for subsequently computing other member material properties such as tensile strength, shear strength, and modulus of elasticity. Refer to Section E-1 Concrete Material Properties for more information regarding the estimation of concrete material properties for existing concrete structures and material sensitivity analyses typically incorporated into risk analyses for reinforced concrete potential failure modes.

Although the specified 28-day strength is often considered to be a conservative estimate for the existing strength of a reinforced concrete member, reference tools are available to estimate adjustments to the 28-day design compressive strength to better represent the existing concrete compressive strength. These reference tools should be used carefully taking into account the original curing conditions, the prevalent environmental conditions over the life of the structure, and the observed condition of the concrete.

If no laboratory data and no design information are available, standard or representative values for material properties based on historical information associated with similar structures of the same era can be used in preliminary structural evaluations with the understanding that there may be significant uncertainty associated with the risk analysis results. The results may or may not be conservative.

Concrete core and laboratory testing may be initiated if the risks or uncertainty from these preliminary analyses are justified. Concrete hydraulic structures are generally built in stages with construction joints. The strength along construction joints is a function of the concrete material properties and also the construction techniques used. If construction joints are poorly constructed they may become unbonded and unable to develop tensile strength or they may have

reduced shear strength. Unfortunately, construction joints are usually positioned at changes in geometry and may be at locations of peak structural demand, for example at the base of a pier.

E-2.1.5 Reinforcement Details

The amount and detailing of the reinforcement, as well as the mode of failure, determines if the member fails in a brittle or ductile manner. Reinforcement development lengths and splice length requirements have changed over the years. Detailing requirements for seismic design have also changed significantly with the evolution of earthquake engineering over the past several decades. Insufficient embedment lengths, splice lengths or hook details can result in sudden pullout failures as shown on Figure E-2-11. Sections will tend to fail in a ductile manner if designed according to relatively recent ACI codes. Appropriate amounts of stirrups will confine areas of damaged concrete and help maintain post-seismic structural integrity. Many hydraulic structures are designed to resist shear based exclusively on concrete shear strength (V_c) and can still behave in a ductile manner; however, lightly reinforced concrete sections, very typical of older massive hydraulic structures, can be overstressed by large earthquakes and yield resulting in flexural nonlinear behavior or uncontrolled inelastic deformations. Comparatively, shear failures tend to be more sudden and brittle than bending or tensile failures.

E-2.1.6 Type and Duration of Loading

Static loads, such as hydrostatic or soil pressures, may act on a member for long durations and are considered sustained loads. There may be no mechanism to stop or resist a structural member in the process of failing if the static loads exceed the capacity of the structure.

Earthquake loads are cyclical and change direction rapidly. As a result, sections may not crack through the member thickness even though the tensile capacity is exceeded for short durations. As shown on Figure E-2-12, the seismic load may not have sufficient duration or have enough significant stress peaks to completely strain a section to failure if it cracks through; or as the section cracks and changes frequency, the response of the structure may change the seismic loads and failure potential. Post-seismic stability must consider the ability of a damaged section to carry static loads.

E-2.1.7 Structural System Considerations

The location and support conditions of the member within the structural system can affect the seismic internal forces acting on the section under investigation and the response of the member. Structural systems that perform well during earthquakes have the ability to dissipate energy through inelastic deformation and have the ability to redistribute loads to elsewhere in the system. In fact, new seismic design details often incorporate the concept of forced plastic hinging at specific locations in the system for the purpose of dissipating energy and redistributing loads within the system. However, hydraulic structures such as spillway walls and piers are generally not highly redundant structures.

E-2.2 Considerations for Analysis Results

While typical hydraulic structures are not necessarily highly redundant structural systems, they do tend to be very large and massive structures. As a result, when evaluating the potential for and degree of overstress, it is important to look at the structure as a whole, and not just focus on a localized maximum value. Judgment may be required to select representative demand/capacity ratios that represent the overall structural component response. Linear elastic finite element analyses may give local overstress values that do not account for additional deflection of a cracked or yielded section. Cracking and straining of reinforcement may result in redistribution of load to other areas of large members. Displacement criteria should be used to evaluate inelastic behavior of reinforced concrete members.

E-2.3 National Codes

National codes like ACI or AASHTO should be used to compute the capacity of existing reinforced concrete sections with caution. If a new structure is being designed, strength reduction factors (ϕ) factors and load factors are applied to develop levels of certainty and factors of safety. In risk analyses for existing reinforced concrete structures, it is desirable to compute the capacity of the sections without ϕ factors and get the “true” expected capacity of the section without extra conservatism. In addition, member demands should be computed without load factors for evaluating risks and determining demand-capacity ratios. Then during the risk analyses team members should consider the condition of the concrete and reinforcement based on the era of construction, severity of the environment, deterioration due to alkali-aggregate

reaction, freeze-thaw, and corrosion. In addition, the strength equations provided in National codes like ACI assume ductile sections with reinforcement details consistent with detailing requirements specified within the code including lap splices, confining reinforcement and anchorage of ties and hooks. As an example, many older spillway bridge deck beams utilize stirrups for shear reinforcement with 90-degree hooks that would be considered inadequate based on the current seismic code requirement for 135-degree hooks as illustrated in Figure E-2-13. In situations where reinforcement detailing is insufficient based on current standards, codes specifically written for evaluation of existing structures such as ASCE 31-Seismic Evaluations of Existing Buildings may be used to determine the capacity of a member with inadequate reinforcement details.

E-2.4 Event tree

An example event tree for the potential failure of a reinforced concrete section is shown in Figure E-2-14. The initiating loads potentially causing the failure are not shown and would typically be included on the front end of the tree to capture the probability of the load occurring.

E-2.1.8 Node 1 – Concrete Stress

Node 1 of the event tree evaluates the probability that the concrete might crack in tension or crush in compression. The static and dynamic tensile strength of concrete is discussed in Section E-1 Concrete Material Properties. The tensile stresses induced on the section under investigation are compared to the estimated range of concrete tensile strength. The tensile strength of the concrete is compared to the magnitude of the computed tensile stresses, the number of excursions above certain values, and the area of the section being overstressed. A few localized excursions of overstress may not significantly crack the concrete over a large enough area to result in structural distress and potentially a brittle failure mechanism.

Tensile stresses in piers, walls, or buttresses are generally caused by moment-induced bending of the section. For evaluation of moment demands the cracking moment M_{cr} must be determined. Specifically, once the tensile strength of the concrete f_t is determined in accordance with Section E-1 Concrete Material Properties, the cracking moment can be computed as:

$$M_{cr} = \frac{f_t \cdot I_g}{y_t} = \text{section cracking moment}$$

Equation E-2-1

where:

f_t = concrete tensile strength

I_g = moment of inertia of the gross concrete section

y_t = distance from the gross concrete section to the extreme tension fiber

If the concrete cracks, branch 1A of Figure E-2-14 is followed and tensile load is transferred from the concrete into the reinforcement. If the concrete does not crack, the concrete carries the tensile load and branch 1B of Figure E-2-14 is followed to Node 3 where the intact shear capacity is evaluated. Most, if not all, hydraulic structures are adequately or under-reinforced because of the massive member sizes. For this reason, concrete crushing is unlikely because the reinforcement will yield well before the concrete crushes. Also, reinforcement is typically placed on both faces. Reinforcement on the compression face may act as compression reinforcement, but is much less effective in resisting bending moments than the tension steel.

E-2.1.9 Node 2 – Reinforcement Response to Bending

Node 2 of the event tree evaluates the possibility of the reinforcing steel yielding given that the concrete cracks. At this node there are two possible paths: either the reinforcement can carry the load and not yield along branch 2A or the reinforcement yields along branch 2B.

The first task for evaluating this node is to identify the yield strength of the reinforcement and the detailing of the reinforcement for the section under investigation. Locate as-built drawings, records of design and construction, the design specifications, and laboratory testing results. If these are not available, Table E-2-1 provides some guidance on possible strengths of reinforcing

steel and concrete strengths can often time be estimated based on historical information associated with similar structures of the same era. .

The second task is to compute the moment (M) and axial (P) capacity of the section represented on a P-M diagram (also referred to as an interaction diagram) as shown on Figure E-2-15.

Almost any reinforced concrete design text book can be used as a reference on how to develop a P-M diagram. A P-M diagram can be used in all cases where buckling of the member is not an issue. However, in cases where the axial load is negligible, it may only be necessary to calculate the moment capacity, or if the moment is negligible only the axial capacity. The P and M demands are plotted on the P-M diagram to determine if the loading exceeds the nominal capacity of the section. During seismic loading, biaxial bending is considered and not just bending along the major and minor axes of the member. If the member is analyzed using finite elements, it may be necessary to convert output stresses to P and M values as shown in Figure E-2-16 which can then be plotted on the P-M diagram. This is especially problematic for finite element models that utilize solid brick elements throughout the member of the section under consideration.

The top graph in Figure E-2-15 shows a section that is slightly overstressed beyond its nominal yield capacity (maximum D/C ratio = 1.25). A D/C ratio of 1.25 in the low axial load region of the diagram indicates the stress in the reinforcement is 1.25 times greater than the yield capacity of the reinforcement. The bottom graph of Figure E-2-15 shows a section that is clearly stressed beyond its nominal yield capacity by a factor of 2.5 (maximum D/C ratio = 2.5).

Figure E-2-17 shows example response curves that may be used at Node 2 as a starting point for estimating the likelihood of various flexural responses of the reinforcement. Similar curves or adjustments to the curves provided on Figure E-2-17 can be developed by the risk analysis team and should account for site-specific conditions and the experience of the risk analysis team members. As stated previously, the demand/capacity ratios referenced in Figure E-2-17 are computed without strength reduction (ϕ) factors and load factors suggested by the ACI code. Specifically, it is desirable to compute the “true” capacity of the members without adding conservatism in the computations. This way the risk teams can make judgments on the strength

of the members and then incorporate factors such as concrete deterioration, age of concrete, era of construction, corrosion, spalling, and environmental issues. For the example flexural yielding section response curves indicated on Figure E-2-17, moment D/C ratios of 1.0 or less for lightly reinforced sections and 1.1 or less for adequately reinforced sections indicate it is very unlikely that the reinforcement has yielded.

The lightly reinforced section curve shown on Figure E-2-17 represents the case where the section is lightly reinforced compared to current code requirements and formation of a fully yielded section (plastic hinge) is possible at smaller demand/capacity ratios and is typical of many older hydraulic structures. Lightly reinforced means that the reinforcement was not adequately designed to carry the induced moment, the amount of steel is less than the minimum required by code (A_{smin}), or the moment capacity of the reinforced section is less than the cracking moment capacity of the concrete. This might be the case for a spillway pier that has only temperature and shrinkage reinforcement along the sides of the pier resisting the moment induced by cross-canyon seismic loads. Or this might be the case when the tensile strength of the concrete alone results in a moment capacity greater than the moment capacity of the reinforced section. This can cause a brittle failure when the concrete cracks and moment demands are suddenly transferred from the concrete to the reinforcement. For such lightly reinforced sections, the section would be expected to fully yield just beyond the expected nominal moment capacity of the section computed as (neglecting compression reinforcement):

$$M_n = A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) = \text{nominal moment capacity} \quad \text{Equation E-2-2}$$

The adequately reinforced section curve represents a section that has reinforcement and detailing generally consistent with current codes. In this case, the formation of a fully yielded section (plastic hinge) is not estimated to be virtually certain until a higher D/C ratio of 1.25 is reached. As indicated on Figure E-2-17, this D/C ratio is consistent with the probable moment strength (M_{pr}) associated with plastic hinging of the flexural member defined in ACI 318 as:

$$M_{pr} = A_s \cdot 1.25f_y \cdot \left(d - \frac{a}{2} \right)$$

Equation E-2-3

= probable moment strength at plastic hinging

where:

$$a := \frac{1.25f_y}{0.85f_c} \cdot b = \text{depth of the compression block}$$

The use of Figure E-2-17 represents the probability that the reinforced concrete section under investigation has transitioned from an elastic section to a fully yielded inelastic section or plastic hinge. The likelihood that the reinforced concrete section is in the elastic range is one minus this value. The probability of steel rupture is not addressed in Figure E-2-17. Rather, displacement criteria are used in subsequent nodes of the event tree to ultimately address the potential for uncontrolled nonlinear displacements including rupture of the reinforcement. Note that there is a strain requirement for yielding reinforcement that has to do with the actual development length (not the computed embedment length, but the transfer length). If this total strain is not reached, yield will not occur and a sudden pullout failure could result as shown in Figure E-2-11. The curves in Figure E-2-17 do not take into account the potential for sudden pullout failure of the reinforcement.

There are several conditions that can influence this node. If the reinforcement has corroded and has lost cross-sectional area, the capacity of the reinforcing steel will be reduced and the reinforcement will yield at a lower load. This may be difficult to detect, but if the face of the concrete is cracked and the environment will subject the structural member to moisture, then this condition could be considered in sensitivity studies. If spalling has occurred that reduces the concrete section of a structural member, this can also reduce the capacity of the member. Finally, if the embedment lengths or lap lengths of the reinforcing bars are not adequate to fully develop the strength of the bars, the reinforcement may debond and fail at a load that is less than what would have occurred with adequate embedment length or lap length.

E-2.1.10 Node 3 – Section Response to Shear

Node 3 of the event tree evaluates the shear failure of the section. Nodes 3A and 3B evaluate the potential for the section to shear given a cracked section but with the reinforcement stressed below the elastic limit. If the section fails in shear the section is then evaluated along branch 3A for kinematic instability (see below). Branches 3C and 3D evaluate the potential for shear failure given a cracked concrete section where the moment steel has yielded. The shear capacity of this section is less than the section in branches 3A and 3B because the concrete section has cracked resulting in a potentially reduced shear capacity and reinforcement that has yielded. Branches 3E and 3F represent the shear strength of a concrete section that has not cracked due to bending and the reinforcement has not yielded. This section has the highest shear capacity of all the Node 3 possibilities because the concrete is intact and the reinforcement has not yielded. If any of the Node 3 sections fail in shear, the branches proceed to evaluating the kinematic instability of the section in Node 5.

The response curve shown on Figure E-2-18 is an example that may be used for estimating likelihood values of shear failure given the demand to capacity (D/C) for shear. However, the risk analysis team should develop their own shear response curve based on the existing condition of the structural members under investigation and the team's expertise and judgment.

Shear stress along a slide plane to be used for computing demand-capacity ratios may be computed in a number of ways. Figure E-2-19 shows an example of shear stresses output from a finite element study. Care must be taken to ensure sufficient mesh density when calculating shear stresses, as a coarse mesh will result in an overly simplified (and low) shear stress distribution. The extent of overstressing over the entire structure is considered.

The shear capacity should be computed considering the height to width ratio of the member, the amount and orientation of shear reinforcement developing a clamping (normal) force, the condition of construction joints, the normal stress acting on the section under investigation, and the amount of yielding of the flexural reinforcement.

For reinforced hydraulic structures that are massive and lightly reinforced, resistance to shear may be evaluated using block sliding methodology similar to what is used for evaluation of sliding for concrete gravity dams. Specifically, recommendations for the shear strength of lightly reinforced massive members are presented by Electric Power Research Institute (EPRI) in Figure E-2-20 for intact structures with fully bonded (uncracked) construction joints, and Figure E-2-21 for cracked (but not severely damaged) structures or structures with unbonded construction joints. The data is intended for large unreinforced or lightly reinforced deep members with height to depth ratios of 4 or less and have horizontal sliding surfaces that are bonded or unbonded. .

The EPRI shear data of Figures E-2-20 and E-2-21 was obtained from tests on mass concrete. The normal force acting on the slide plane in relation to the failure envelope determines the shear capacity. A straight line fit through the data is expressed as cohesion (shear strength at zero normal stress) and friction angle. The intercept of the line at zero normal stress is called the “true” cohesion for a bonded member and called the “apparent” cohesion for an unbonded member. When using the EPRI data, if the concrete section has cracked, the true (bonded) cohesion of the concrete should no longer be used. In this case, the shear strength is nonlinear, passing through zero strength at zero normal stress as illustrated by the red dashed line on Figure E-2-21. Apparent cohesion should be used with extreme care and depends on the normal stress acting on the slide surface. The normal stress on a surface should be relatively constant and relatively easy to compute by resolving the forces acting on the section (including weight, driving force, and tensile forces in the steel) normal and parallel to the crack (or potential sliding plane), and using the normal force to estimate the range of expected shear strengths. The effective friction angle at low normal stress levels, typical of massive hydraulic structures, is likely to be high (in excess of 45 degrees). For higher values of normal stress, a combination of apparent cohesion and lower friction angle may be used if it is impractical to use a nonlinear failure envelope.

It should be noted that many concrete sections associated with hydraulic structure are much more massive than representative sections considered in the ACI code. As such, shear friction plays a more significant role in the shear strength of a member as well as the sliding stability (Node 5).

Shear frictional resistance is not only produced by the weight of the concrete on the cracked surface, but also by tensioning of the steel during shear dilation of the rough cracked surface, which results in additional normal stress and frictional resistance across the crack. Estimates of roughness can be used to help establish the amount of crack opening and steel tensioning that would occur during shearing of a rough crack. Shear friction for unreinforced members is computed using the following equation:

$$SF = (N - U)\mu + CA \qquad \text{Equation E-2-4}$$

Where:

SF = shear resistance

N = Normal force on the sliding plane

U = Uplift forces along sliding plane

μ = Friction coefficient (tangent of the friction angle)

C = Cohesion (apparent cohesion)

A = Area of slide surface

The normal force generated from tensioning of the reinforcing steel can be added to the normal force in this equation.

The ACI code provides considerable guidance for shear strength of reinforced concrete members. The ACI code can be used to calculate the shear demand capacity ratios, but care must be taken to use the equations that are appropriate to the case being studied. This requires careful consideration of the orientation of the crack, the orientation of the reinforcing steel relative to the crack, and the ratio of steel area to concrete area. The ACI equations include:

$$V_n = V_c + V_s = \text{nominal shear capacity}$$

Equation E-2-5

where:

V_c = concrete shear strength

V_s = reinforcement shear strength

According to ACI, shear friction reinforcement calculations do not apply when a diagonal tension shear failure, typical of most cast-in-place concrete members, develops. Rather ACI designates that shear friction reinforcement should be included in shear strength calculations for shear transfer across a given plane such as an existing or potential crack, an interface between dissimilar materials, or an interface between two concretes cast at different times. In addition, shear friction reinforcement should be supplemental to the primary flexural reinforcement. In some cases, the ACI code may provide a conservative estimate of shear strength and for more massive structures, such as spillway piers, using the sliding friction approach and the curves in Figures E-2-20 and E-2-21 may be more appropriate.

In the case where a member is severely damaged by many cycles of loading, it may not be appropriate to apply the shear friction approach as described above. The concrete may be so badly damaged that the steel is ineffective and the concrete may be nearly rubble. Judgment is required to determine if this is likely, and if so, what the shear strength is likely to be.

Caltrans provides guidance to compute the shear strength for reinforced concrete members with yielded moment reinforcement and less than adequate shear reinforcement. However, this criterion was developed largely for tied columns with adequate confining reinforcement, and should be applied with caution to hydraulic structures. In this case, the concrete core remains intact because the tie reinforcement limits spalling.

Failure can progress in a rapid and brittle manner if the shear capacity is exceeded. Figure E-2-18 is provided as an example for estimating failure likelihoods for a potential shear failure scenario. In the example shear response curve of Figure E-2-18, the shear failure probabilities are estimated to be approximately neutral for shear D/C ratios of 1.0, likely when shear D/C

ratios approach 1.1, and virtually certain when shear D/C ratios approach 1.2. Similar to the flexural response curve of Node 2, shear response curves should be developed on a case-by-case basis considering the computed shear D/C ratio, the estimated amount of reinforcement that has yielded, the frictional resistance produced by the member weight, the orientation of the failure plane, the strength of the concrete, the strength of the reinforcement, the magnitude of shear force, the static or cyclical nature of the loading, and the duration of loading.

E-2.1.11 Node 4 – Displacement Criteria

Node 4 of the event tree evaluates the displacement of the section if the reinforcement has yielded. Even if the section has adequate shear capacity, branch 4A of Figure E-2-14 represents the potential for the section to fail by uncontrolled inelastic displacement that could lead to subsequent failure of the spillway gates or collapse of the wall. If the section meets the yield displacement criteria and the shear criteria of branch 3D, then the structure may be considered viable. For an event tree dealing exclusively with static loading, this node should generally be eliminated from the event tree since uncontrolled inelastic deflections are virtually certain to occur under static moment demands that exceed the yield moment capacity of the section unless the member is part of a highly redundant structural system (generally not the case for typical hydraulic structures).

$$\delta \leq 2 \text{ to } 3\delta_{\text{yield}}$$

Equation E-2-6

where:

δ = computed deflection

δ_{yield} = yield displacement

More specifically, Node 4 of the event tree evaluates if the probability that inelastic deflections of the structure, resulting from the yielded cross section of Node 2, progress to the point of structural instability. Research at the University of Illinois at Champagne-Urbana by Dr. Mete Sozen has shown that reinforced sections can deform beyond the yield displacement of the section and still remain stable (Gulkan and Sozen, 1974; Otani and Sozen, 1974; Nuss, et al.,

1994). The yield displacement is the amount of displacement a section needs to deflect to yield the reinforcement. Figure E-2-22 shows the calculations for the yield displacement of a reinforced concrete cantilever beam. As shown on Figure E-2-22, the yield displacement is different for a concentrated load compared to a uniform load. Therefore, the yield displacement must be calculated for specific loads that cause yielding of the reinforcement based on the configuration of the entire structural system. The yield deflection occurs when the reinforcement first experiences yield tension, assuming the load transfers directly through the crack. However, the load needs to be developed within the reinforcement by creating strain over the development length and, as a result, the actual yield deflection will generally be greater than those computed using equations such as those presented in Figure E-2-22.

The displacements of the member in a damaged state with cracked concrete and yielded reinforcement will be greater than displacements if all material remains intact. The preferred method to compute the damaged displacement is using a non-linear finite element model. A completely cracked through concrete section or a completely unbonded construction joint can be modeled with contact surfaces and the reinforcement can be modeled with shell elements or truss members using non-linear steel properties as shown on the elastic-plastic stress-strain curve of Figure E-2-23. As computing capacity continues to increase, the level of sophistication and degree of non-linearity that can be modeled has significantly improved the ability to predict failure of hydraulic structures.

Computing the damaged displacement using traditional methods or using a linear-elastic finite element model is less accurate and introduces a high level of uncertainty associated with predicting this node of the event tree. The computation of the expected deflection using traditional methods may more appropriately consider a moment of inertia based on a cracked section (I_{cracked}) instead of an uncracked section (I). When linear elastic analyses are used, the modulus of elasticity of the concrete in the damaged zone is typically reduced to one-third to one-half the original value as suggested by Dr. Mete Sozen (Nuss, et al., 1994). For typical reinforced sections, research performed by Dr. Mete Sozen, indicates calculated displacements for a damaged member of 2 to 3 times the yield displacement are unlikely to very unlikely to result in uncontrolled nonlinear displacements; displacements of 3 to 4 times the yield

displacement are neutral in terms of resulting in uncontrolled nonlinear displacements; and displacements of 4 to 5 times the yield displacement or greater are virtually certain to result in uncontrolled nonlinear displacements. Figure E-2-24 provides response curves consistent with these values that may be used for risk analyses. However, great care should be used in evaluating yield displacements based on structural analysis results. Specifically, the ability of a concrete structural system to perform well inelastically is also highly dependent on member reinforcement details such as embedment length, splice length, hook details and confinement reinforcement, which are details that are generally not included in finite element models.

E-2.1.12 Node 5 – Kinematic Instability

Node 5 of the event tree evaluates the kinematic stability of the section. Kinematic failure is the sliding or toppling of a concrete section that is completely independent or essentially separated from the main concrete structure or in a yielded configuration. At this point, the concrete has cracked through; the section has failed in shear or yielded in flexure severely deforming the reinforcement; and the section is effectively acting as an independent concrete member separate from the connecting or supporting member or as a yielded structure requiring consideration of P- δ effects. In either case, the damaged member can either slide or topple depending on the magnitude and duration of the applied loads and the side restraint.

As a result, Node 5 represents the kinematic failure of the damaged member that has either: 1) failed in shear, forming an independent concrete block separate from the rest of the structure, 2) failed due to uncontrolled displacements of the yielded member (most likely during a seismic event), or 3) failed due to post-seismic instability of the yielded member. The evaluation of this node considers the possibility that the damaged section can remain stable or sustain minor movements that do not adversely affect its ability to retain the reservoir rather than slide or topple resulting in an uncontrolled release of the reservoir. Kinematic failures caused by static loads (e.g. post-earthquake) that exceed the resistance of the damaged member are generally considered virtually certain because there may be no mechanism to stop the movements. Kinematic movement caused by seismic loads may not be sufficient to fail the structure if the post-earthquake loads are less than the resistance of the damaged member. In addition, it is possible that surrounding appurtenant features or geometric limitations could preclude kinematic

instability. Otherwise, sufficient duration and magnitude of earthquake shaking is required to fail a structure. The amount of sliding of a separated concrete block can be computed with nonlinear finite element analyses or Newmark analyses. Changes in uplift pressures on potential sliding planes as a result of earthquake movement need to be considered. In addition, the risk analysis team needs to consider that the shear resistance (friction) reduces the farther a member slides and eventually reaches the residual strength of the material as shown on Figure E-2-25 because the roughness of the slide plane gets ground down. Post-seismic stability may need to consider new static loads based on the yielded shape of the member in terms of second-order P- δ analyses as illustrated in Figure E-2-26.

E-2.5 References

Concrete Reinforcing Steel Institute, "Evaluation of Reinforcing Bars in Old Reinforced Concrete Structures," Engineering Data Report Number 48, 2001.

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**STABILITY CONSIDERATIONS
FOR
REINFORCED AND UNREINFORCED CONCRETE WALLS
AND SPILLWAY PIERS**

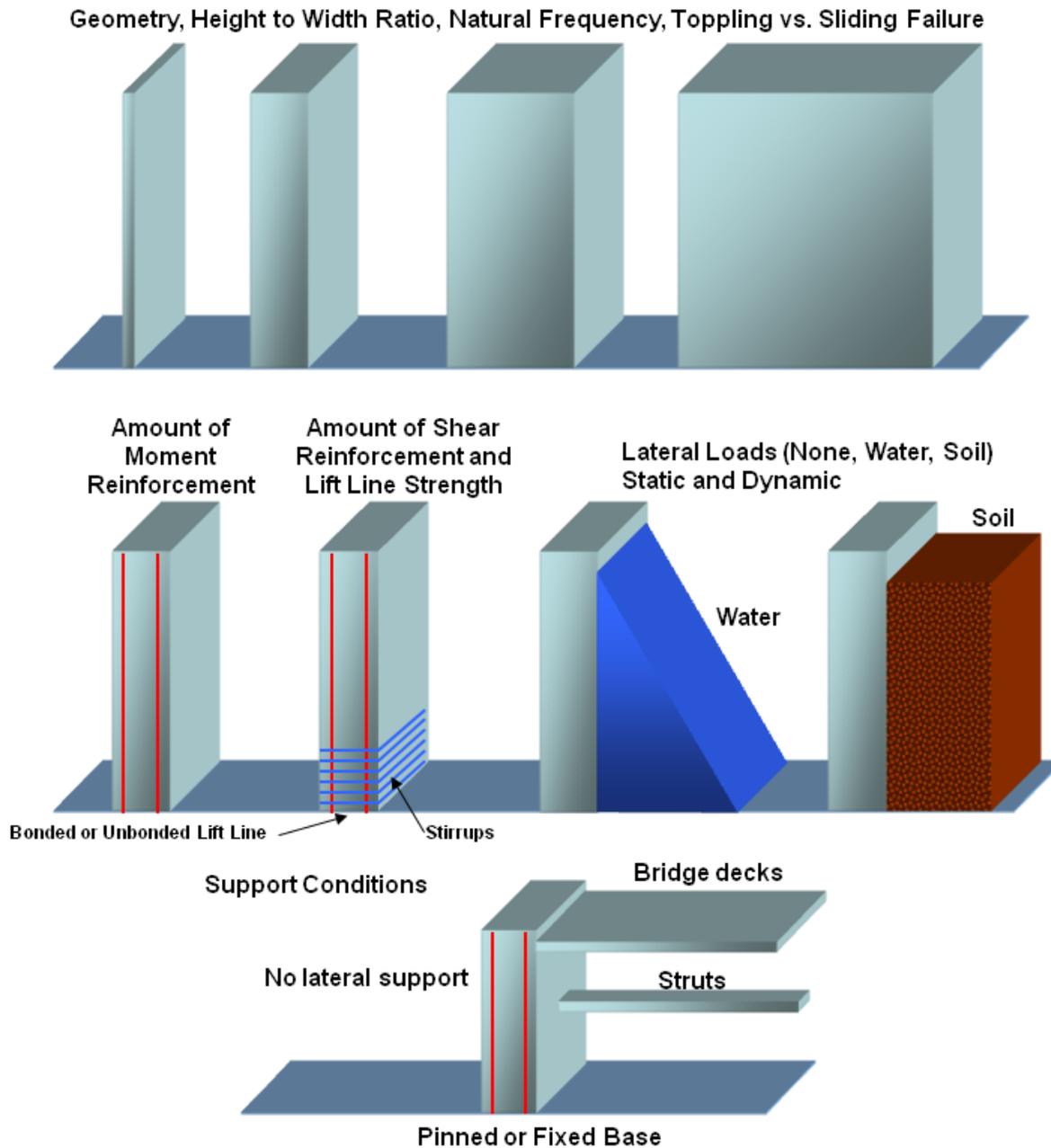


Figure E-2-1 Typical Geometry and Support Conditions for Reinforced Concrete Members in Hydraulic Structures.



Glendo Dam Chute Walls



Stampede Dam Control Structure



Stampede Dam Stilling Basin

Figure E-2-2 Typical Spillway Wall Configurations.



**Paducah, KY Floodwall
Ohio River 2011**



**Sunbury, PA Floodwall
Susquehanna River
Hurricane Agnes 1972**



**New Orleans Floodwall
Hurricane Katrina 2005**

Figure E-2-3 Typical Floodwall Configurations.



Canyon Ferry Dam Gate Piers



Minidoka Dam Canal Headworks Gate Piers



Glen Canyon Dam Gate Piers

Figure E-2-4 Typical Spillway Pier Configurations.



Coolidge Dam Thick Buttress Construction



Stony Gorge Dam Thin Buttress Construction

Figure E-2-5 Typical Dam Buttress Configurations.

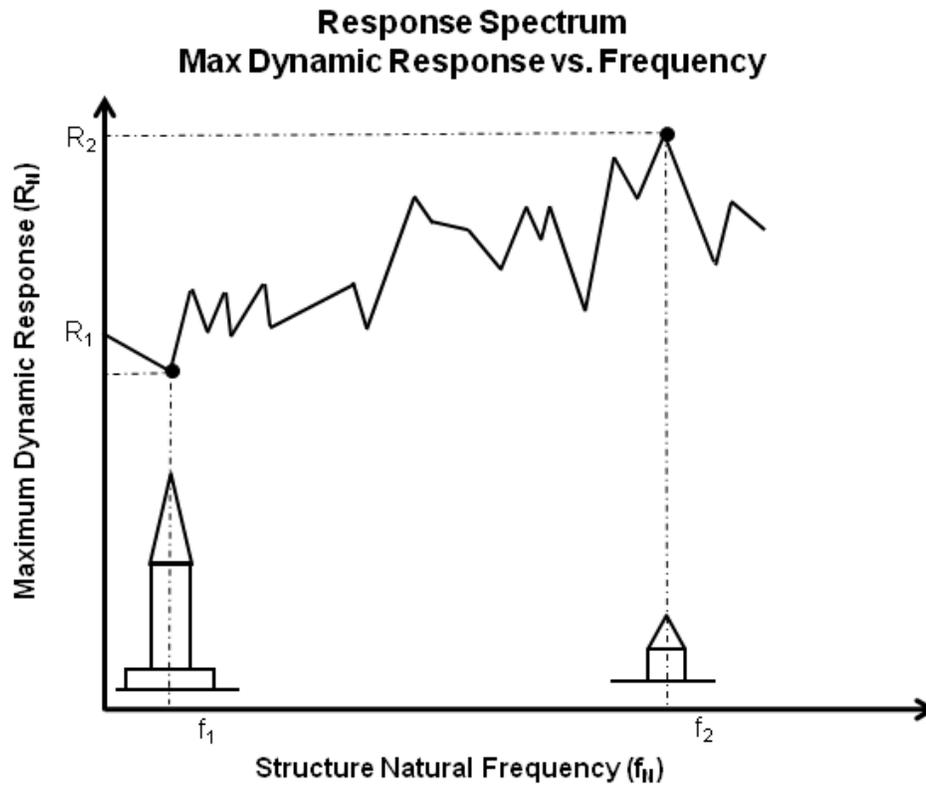


Figure modified from ASCE *Seismic Design and Performance of Building Structures*, 2008

Figure E-2-6 Typical Dynamic Response Spectrum Plot.

NATURAL FREQUENCIES OF VARIOUS SYSTEMS
SLENDER BEAMS [R2]

SLENDER BEAMS (GIVEN)

$$\underline{W} := 10 \cdot \text{lb} \quad \underline{M} := \frac{W}{g} \quad M = 0.0259 \cdot \text{lb} \cdot \frac{\text{s}^2}{\text{in}} \quad \text{Weight and Mass of block}$$

$$\underline{W}_b := 100 \cdot \text{lb} \quad \underline{M}_b := \frac{W_b}{g} \quad M_b = 0.0026 \cdot \text{lb} \cdot \frac{\text{s}^2}{\text{in}} \quad \text{Weight and mass of beam}$$

$$\underline{L} := 120 \cdot \text{ft} \quad L = 1440 \cdot \text{in}$$

$$\underline{E} := 3000000 \cdot \frac{\text{lb}}{\text{in}^2}$$

$$\underline{b} := 12 \cdot \text{in} \quad \underline{h} := 12 \cdot \text{in}$$

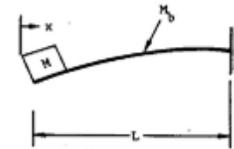
$$\underline{A} := b \cdot h \quad A = 144 \cdot \text{in}^2 \quad I := \frac{b \cdot h^3}{12} \quad I = 1728 \cdot \text{in}^4$$

CANTILEVER SLENDER BEAMS (W/ AND W/O MASS) WITH END LUMPED MASS

$$\underline{\delta} := \frac{W \cdot L}{A \cdot E} \quad \delta = 0 \cdot \text{in}$$

$$\underline{f} := \left(\frac{1}{2 \cdot \pi} \right) \cdot \sqrt{\frac{3 \cdot E \cdot I}{L^3 \cdot (M + 0.24 \cdot M_b)}} \quad f = 2.23 \cdot \text{Hz}$$

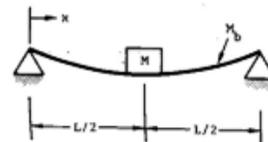
2. Mass, Cantilever



SIMPLE SLENDER BEAM (W/ AND W/O MASS) WITH CENTER LUMPED MASS

$$\underline{f} := \left(\frac{2}{\pi} \right) \cdot \sqrt{\frac{3 \cdot E \cdot I}{L^3 \cdot (M + 0.49 \cdot M_b)}} \quad f = 8.8 \cdot \text{Hz}$$

4. Center Mass, Pinned-Pinned Beam



FIXED END SLENDER BEAM (W/ and W/O MASS) WITH CENTER LUMPED MASS

$$\underline{f} := \left(\frac{4}{\pi} \right) \cdot \sqrt{\frac{3 \cdot E \cdot I}{L^3 \cdot (M + 0.37 \cdot M_b)}}$$

$$f = 17.7 \cdot \text{Hz}$$

22. Center Mass, Clamped-Clamped Beam

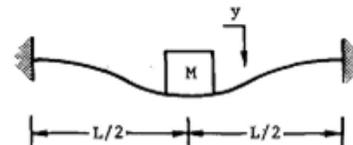


Figure E-2-7 Equations for Calculating Natural Frequency of Beams Including a Lumped Mass.

NATURAL FREQUENCIES OF VARIOUS SYSTEMS
BEAMS [R2]

Given

$$\underline{W} := 0 \cdot \text{lb} \quad \underline{M} := \frac{W}{g} \quad M = 0 \cdot \text{lb} \cdot \frac{\text{s}^2}{\text{in}} \quad \text{Weight and Mass of block}$$

$$\underline{W_b} := 344986 \cdot \text{lb} \quad \underline{M_b} := \frac{W_b}{g} \quad M_b = 0.0026 \cdot \text{lb} \cdot \frac{\text{s}^2}{\text{in}} \quad \text{Weight and mass of beam}$$

$$\underline{L} := 276 \cdot \text{ft} \quad L = 3312 \cdot \text{in}$$

$$\underline{w_b} := \frac{W_b}{L} \quad w_b = 1249.9 \cdot \frac{\text{lb}}{\text{ft}} \quad \underline{m_b} := \frac{w_b}{g} \quad m_b = 38.85 \cdot \text{lb} \cdot \frac{\text{s}^2}{\text{ft}^2} \quad \text{Weight and mass per length of beam}$$

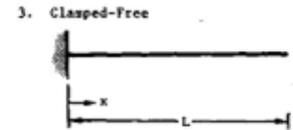
$$\underline{E} := 3000000 \cdot \frac{\text{lb}}{\text{in}^2}$$

$$\underline{b} := 12 \cdot \text{in} \quad \underline{h} := 100 \cdot \text{in}$$

$$\underline{A} := b \cdot h \quad A = 1200 \cdot \text{in}^2 \quad \underline{I} := \frac{b \cdot h^3}{12} \quad I = 1 \times 10^6 \cdot \text{in}^4$$

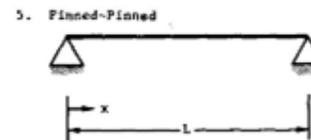
CANTILEVER BEAM (FIXED - FREE) $\lambda := 1.87510407$

$$\underline{f} := \left(\frac{\lambda^2}{2 \cdot \pi \cdot L^2} \right) \cdot \sqrt{\frac{E \cdot I}{m_b}} \quad f = 0.170 \cdot \text{Hz} \quad \underline{T} := \frac{1}{f} \quad T = 5.878 \cdot \text{s}$$



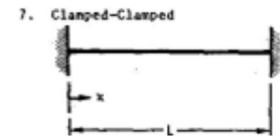
SIMPLE BEAM (PINNED - PINNED) $\lambda := \pi$

$$\underline{f} := \left(\frac{\lambda^2}{2 \cdot \pi \cdot L^2} \right) \cdot \sqrt{\frac{E \cdot I}{m_b}} \quad f = 0.48 \cdot \text{Hz} \quad \underline{T} := \frac{1}{f} \quad T = 2.094 \cdot \text{s}$$



BEAM (FIXED - FIXED) $\lambda := 4.73004074$

$$\underline{f} := \left(\frac{\lambda^2}{2 \cdot \pi \cdot L^2} \right) \cdot \sqrt{\frac{E \cdot I}{m_b}} \quad f = 1.08 \cdot \text{Hz} \quad \underline{T} := \frac{1}{f} \quad T = 0.924 \cdot \text{s}$$



CANTILEVER BEAM (FIXED - PINNED) $\lambda := 3.92660231$

$$\underline{f} := \left(\frac{\lambda^2}{2 \cdot \pi \cdot L^2} \right) \cdot \sqrt{\frac{E \cdot I}{m_b}} \quad f = 0.75 \cdot \text{Hz} \quad \underline{T} := \frac{1}{f} \quad T = 1.341 \cdot \text{s}$$

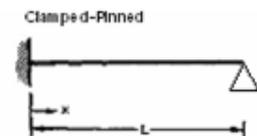


Figure E-2-8 Equations for Calculating Natural Frequency of Beams.

NATURAL FREQUENCIES OF VARIOUS SYSTEMS
GRAVITY WALLS [R1]

GRAVITY WALLS [R1]

$$H := 30 \text{ ft}$$

$$S := 0.7$$

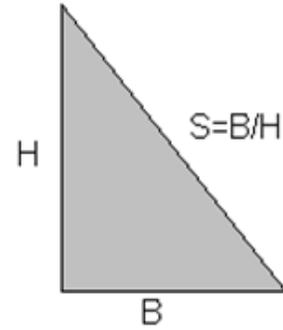
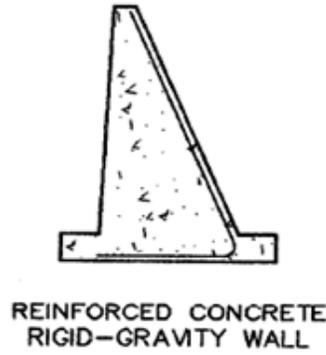
$$B := H S$$

$$F := 0.000425 \frac{\text{sec}}{\text{ft}}$$

$$T := \frac{F H^2}{B}$$

$$f := \frac{1}{T}$$

$$B = 21 \text{ ft}$$



$$T = 0 \text{ s}$$

$$f = 54.9 \text{ Hz}$$

CANTILEVER WALL FIXED AT BASE ($B/H < 0.5$) [R1]

$$H := 276 \text{ ft}$$

$$B := 8.33 \text{ ft}$$

$$F := 0.000643 \frac{\text{sec}}{\text{ft}}$$

$$T := \frac{F H^2}{B}$$

$$f := \frac{1}{T}$$

$$T = 5.8$$

$$f = 0.17$$

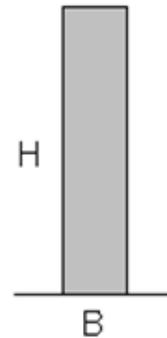
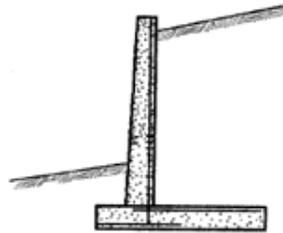


Figure E-2-9 Equations for Calculating the Natural Frequency of a Gravity Wall and a Cantilever Wall.

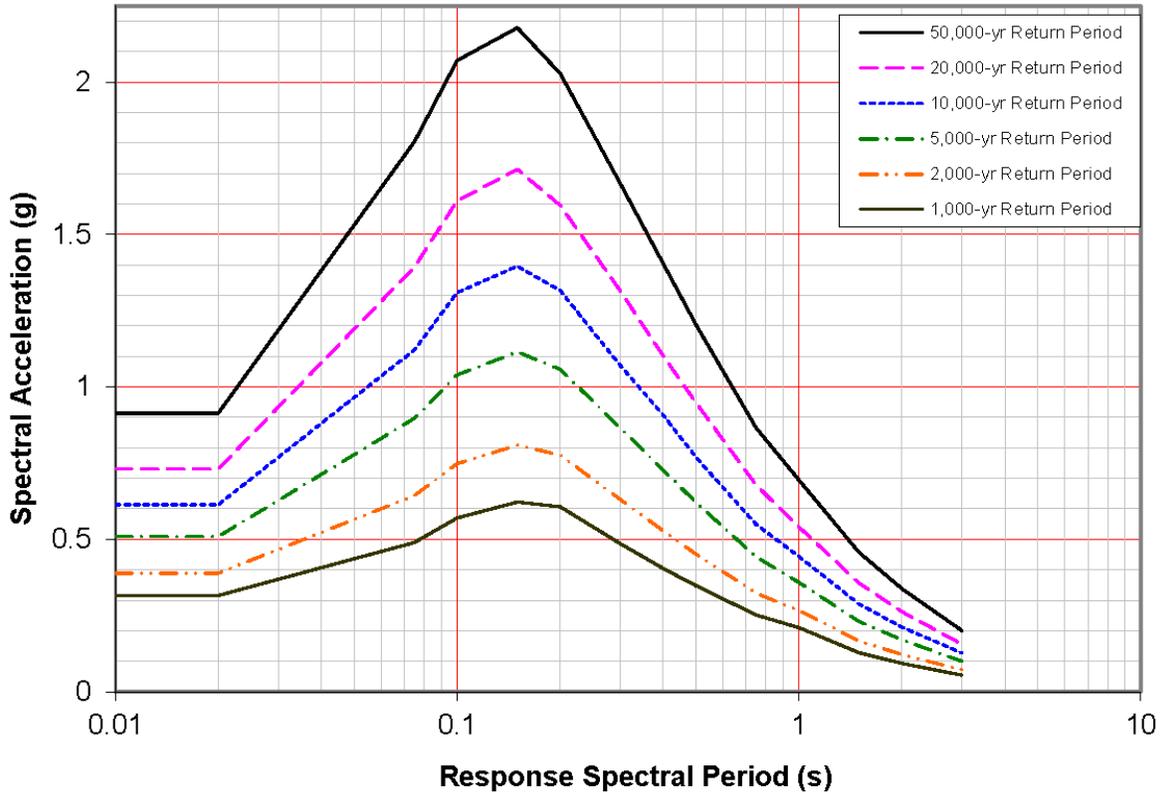


Figure E-2-10 Uniform hazard spectra (UHS) for earthquake return periods between 1,000 and 50,000 years.



Embedment/Splice Length



Splicing of Transverse Reinforcement and Hook Ends



Shear Reinforcement

Figure E-2-11 Inadequate Reinforcement

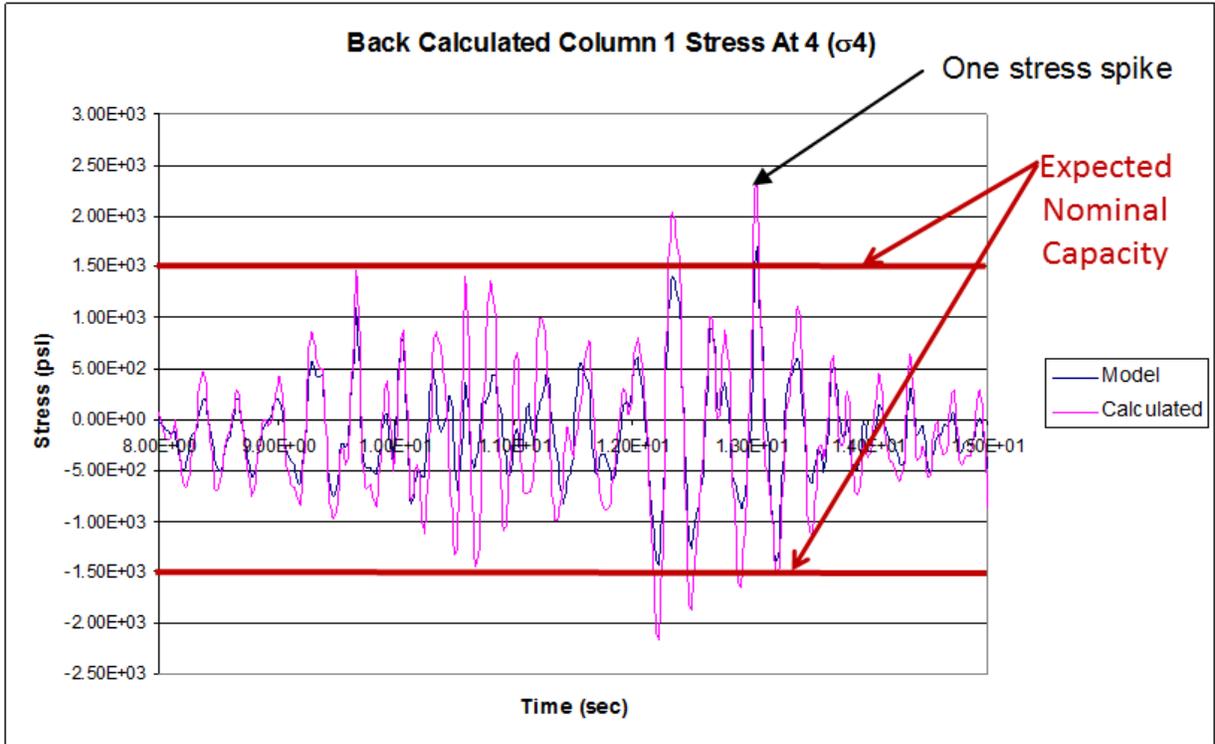


Figure E-2-12 Stress Time-History Results at the Base of a Superstructure for a 50,000-year Earthquake.

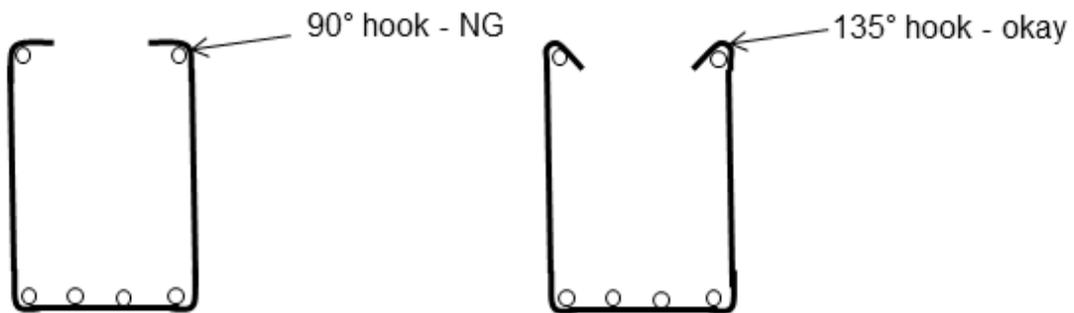


Figure E-2-13 Hook Details Comparing Seismically Unacceptable and Seismically Acceptable Stirrup Details.

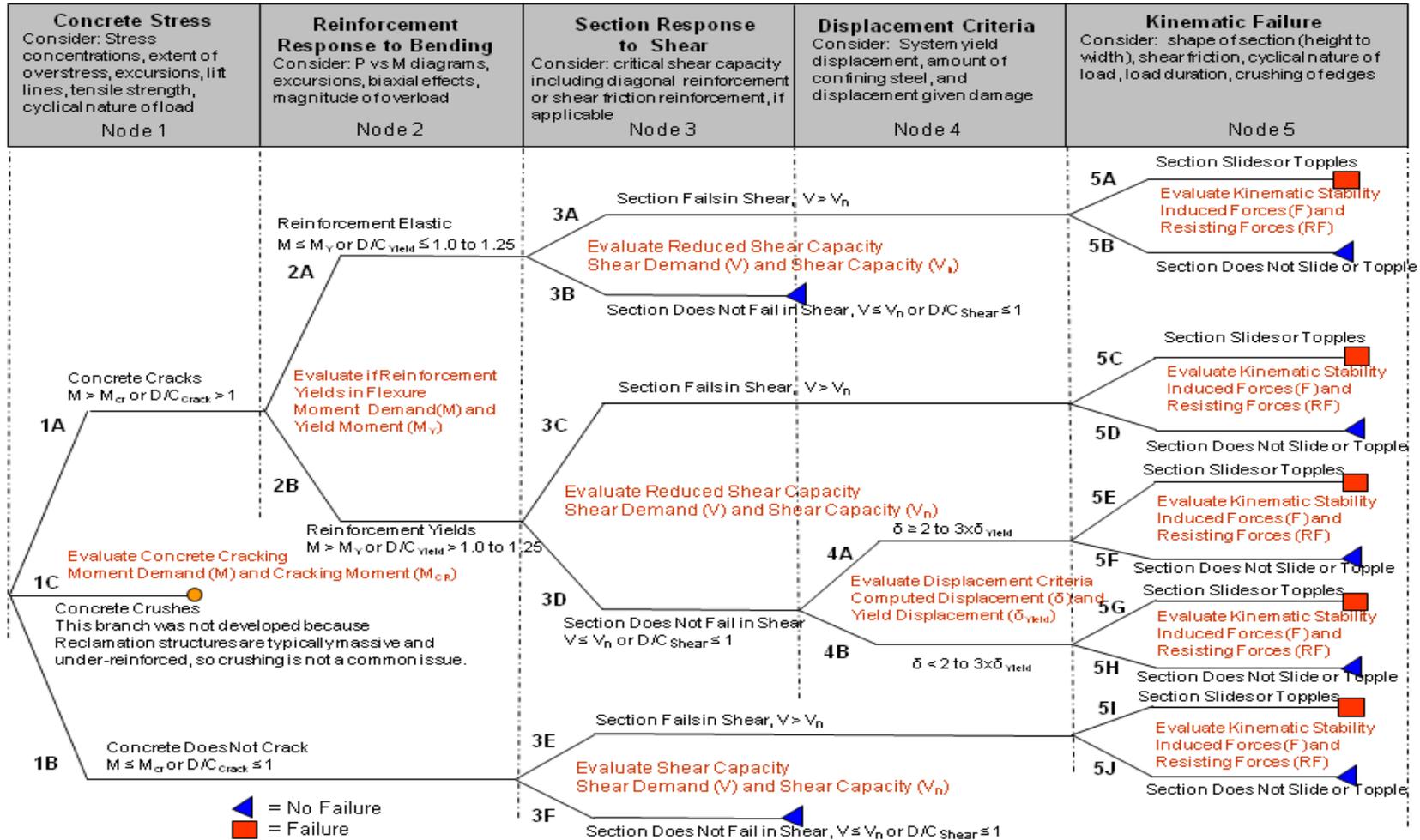
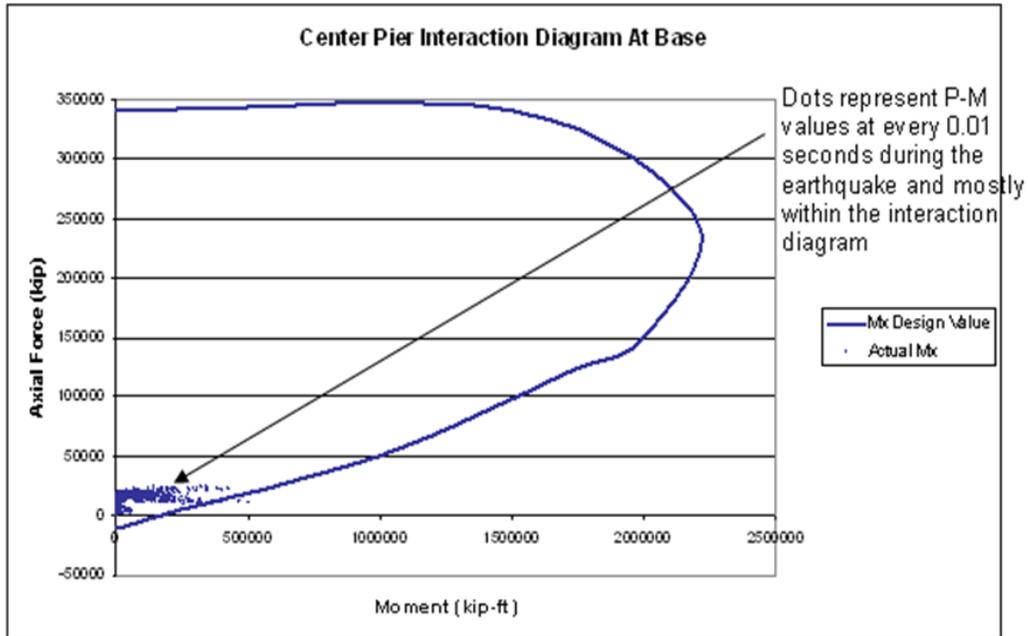
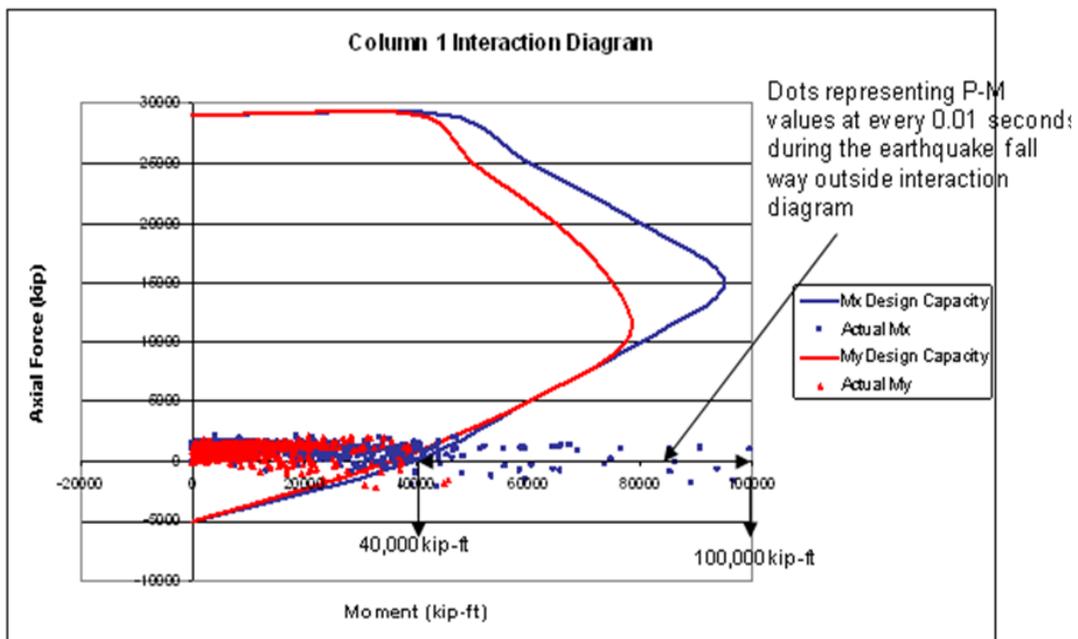


Figure E-2-14 Example Event Tree for Failure of a Reinforced Concrete Member.



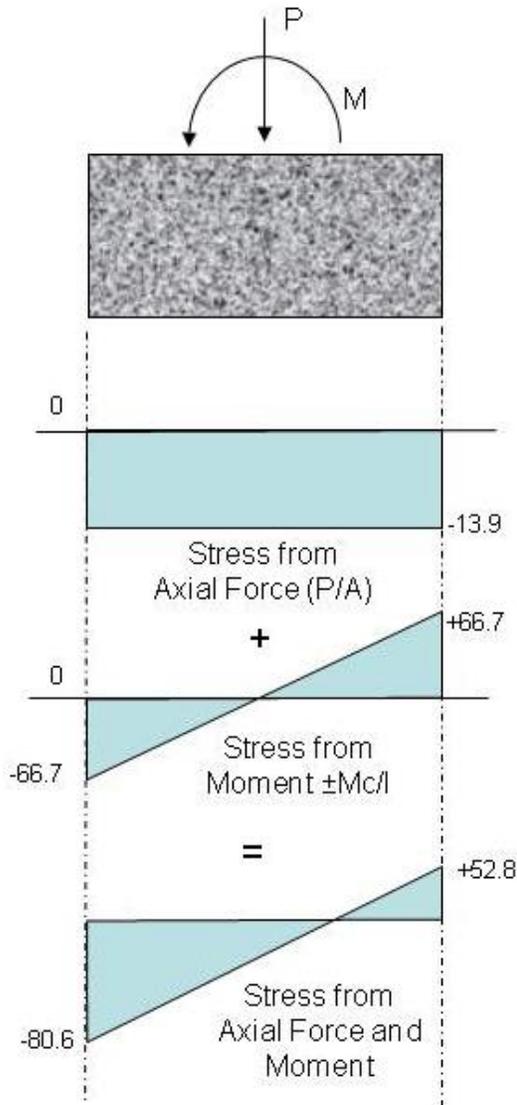
Maximum Demand to Capacity ratio is 1.25 (500,000 / 400,000) meaning steel is stressed 1.25 times beyond its yield



Maximum Demand to Capacity ratio is 2.5 (100,000 / 40,000) meaning steel is stressed 2.5 times beyond its yield

Figure E-2-15 example of a Moment Demand-Capacity Evaluation Using an Axial Force (P) vs. Moment (M) Interaction Diagram Based on Results from a Time History Analysis.

Converting from
Axial Force (P) and Moment (M) to Stresses
and Visa Versa



Given: P, M, and Section Properties

$$P := -10000 \text{ lb}$$

$$M := 40000 \text{ lb} \cdot \text{ft}$$

$$b := 1 \text{ ft}$$

$$h := 5 \text{ ft}$$

$$A := b \cdot h \quad A = 5 \text{ ft}^2$$

$$I := \left(\frac{1}{12}\right) \cdot b \cdot h^3 \quad I = 10.417 \text{ ft}^4$$

$$c := \frac{h}{2} \quad c = 2.5 \text{ ft}$$

Compute Stresses at Points P1 and P2

$$\sigma_1 := \left(\frac{P}{A}\right) + \left(\frac{M \cdot c}{I}\right) \sigma_1 = 52.778 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_2 := \frac{P}{A} - \frac{M \cdot c}{I} \quad \sigma_2 = -80.556 \frac{\text{lb}}{\text{in}^2}$$

Stress Caused by Axial Force Only

$$\frac{P}{A} = -13.889 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{ave}} := \frac{(\sigma_1 + \sigma_2)}{2} \quad \sigma_{\text{ave}} = -13.889 \frac{\text{lb}}{\text{in}^2}$$

Stress Cause by Moment Only

$$\frac{M \cdot c}{I} = 66.667 \frac{\text{lb}}{\text{in}^2}$$

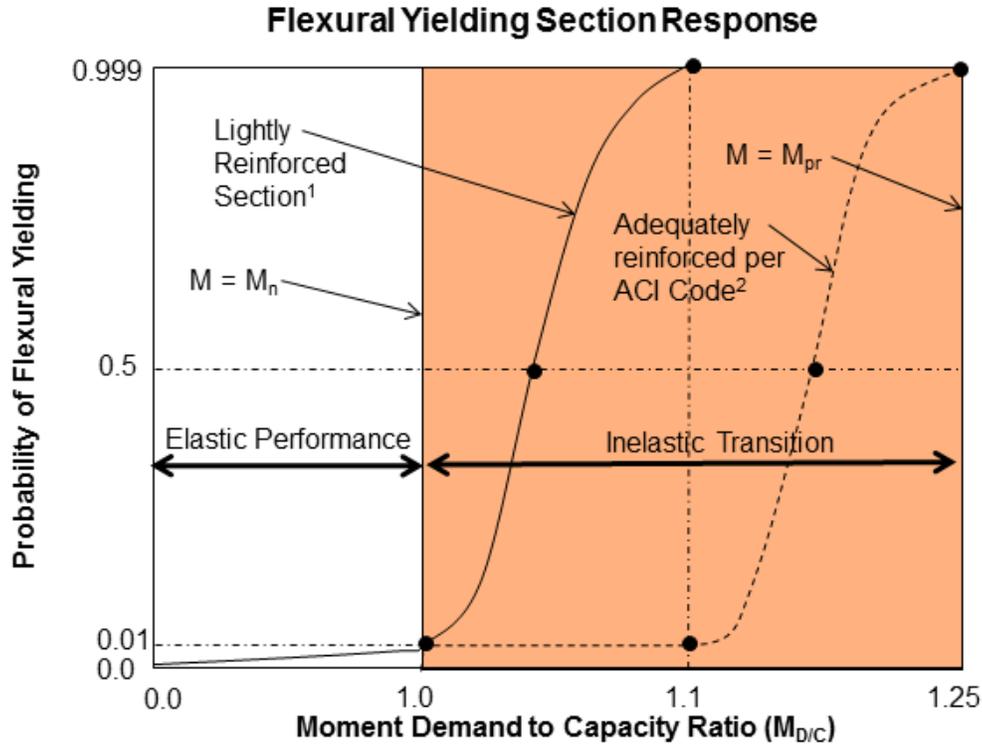
$$\sigma_m := \frac{(\sigma_1 - \sigma_2)}{2} \quad \sigma_m = 66.667 \frac{\text{lb}}{\text{in}^2}$$

Compute P and M Given Stresses

$$P := \frac{(\sigma_1 + \sigma_2)}{2} \cdot A \quad P = -10000 \text{ lb}$$

$$M := \left[\frac{(\sigma_1 - \sigma_2)}{2}\right] \cdot \frac{I}{c} \quad M = 40000 \text{ ft} \cdot \text{lb}$$

Figure E-2-16 Conversion of Force and Moment to Stress.



Notes:

1. Lightly reinforced section is defined as a section with $A_s < A_{s(min)}$ in accordance with current ACI 318 code provisions. This represents a condition that could result in a brittle failure when the cracking moment of the unreinforced concrete section computed using its modulus of rupture exceeds the flexural strength of the reinforced concrete section using cracked section analysis.
2. An adequately reinforced section is defined as a section with $A_s \geq A_{s(min)}$ in accordance with current ACI 318 code provisions. In addition, $\rho \leq 0.75\rho_b$ to ensure a ductile flexural failure mechanism is achievable.
3. Nominal moment capacity (M_n) and probable moment strength (M_{pr}) are computed without strength reduction factors (ϕ). Specifically, M_n and M_{pr} are computed as:

$$M_n = A_s f_y (d - a/2) \text{ (rectangular section – tension reinforcement only)}$$

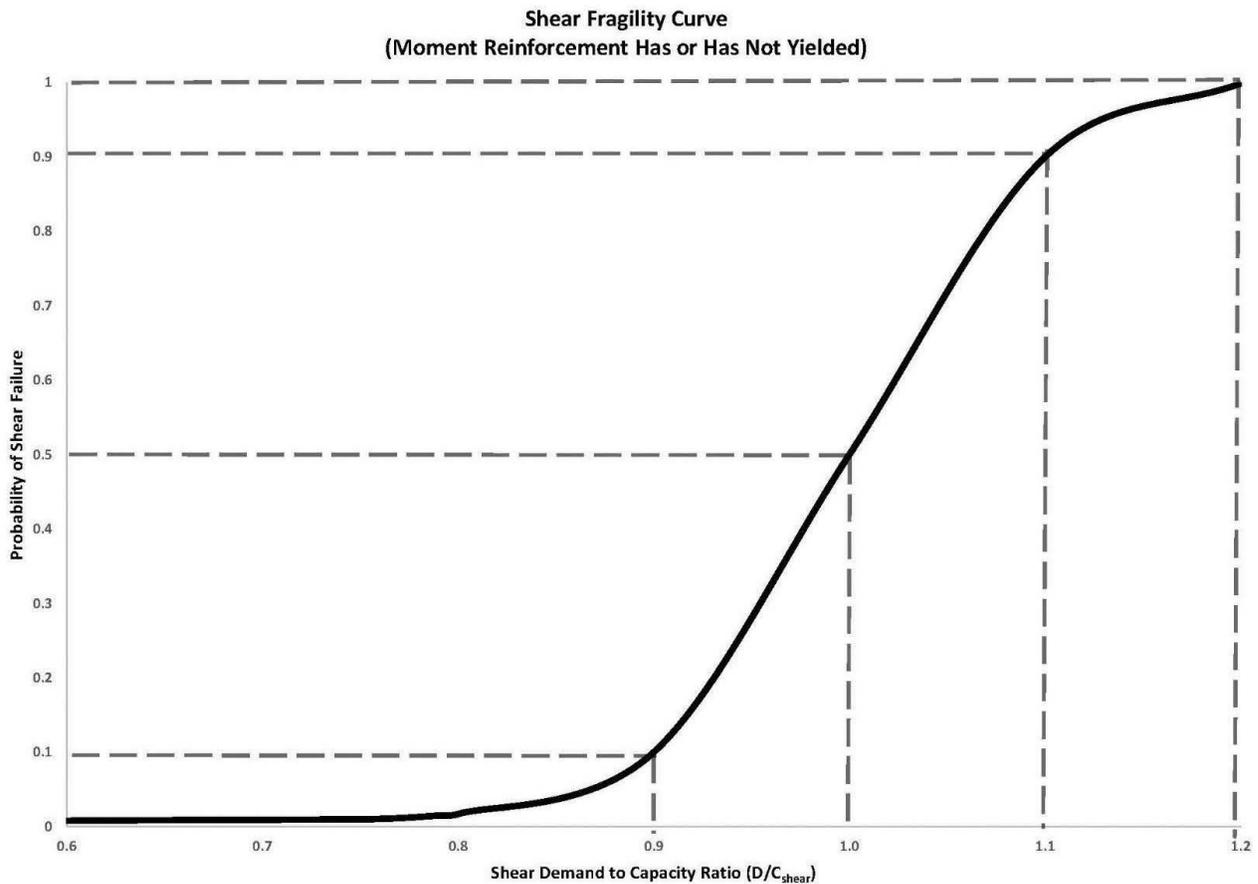
$$\text{where } a = A_s f_y / 0.85 f'_c b$$

$$M_{pr} = A_s (1.25 f_y) (d - a/2) \text{ (rectangular section – tension reinforcement only)}$$

$$\text{where } a = (1.25 f_y) / 0.85 f'_c b$$

5. Moment demands should not include load factors.
6. For evaluation of axially loaded members, P-M interaction diagrams should be utilized.

Figure E-2-17 Flexural Yielding Response Curves.



Notes:

1. This fragility curve applies if the moment reinforcement has or has not yielded (Event Tree Node 2) and the concrete may or may not be cracked (Event Tree Node 1). The idea is that the shear capacity is computed given the condition of the moment reinforcement, the adequacy of the shear reinforcement, and the condition of the lift line or construction joint.
2. No ϕ factors or load factors are applied when computing the D/C ratio for shear.
3. Shear failure (sliding) is generally considered to occur when the D/C is greater than 1.0; however, shear failure may not actually occur at a D/C = 1.0 given the uncertainties. Since shear failure can be a sudden and brittle type failure, failure is judged to be very likely when the D/C ratio in shear is computed to be greater than 1.1.

Figure E-2-18 Example Shear Fragility Curve.

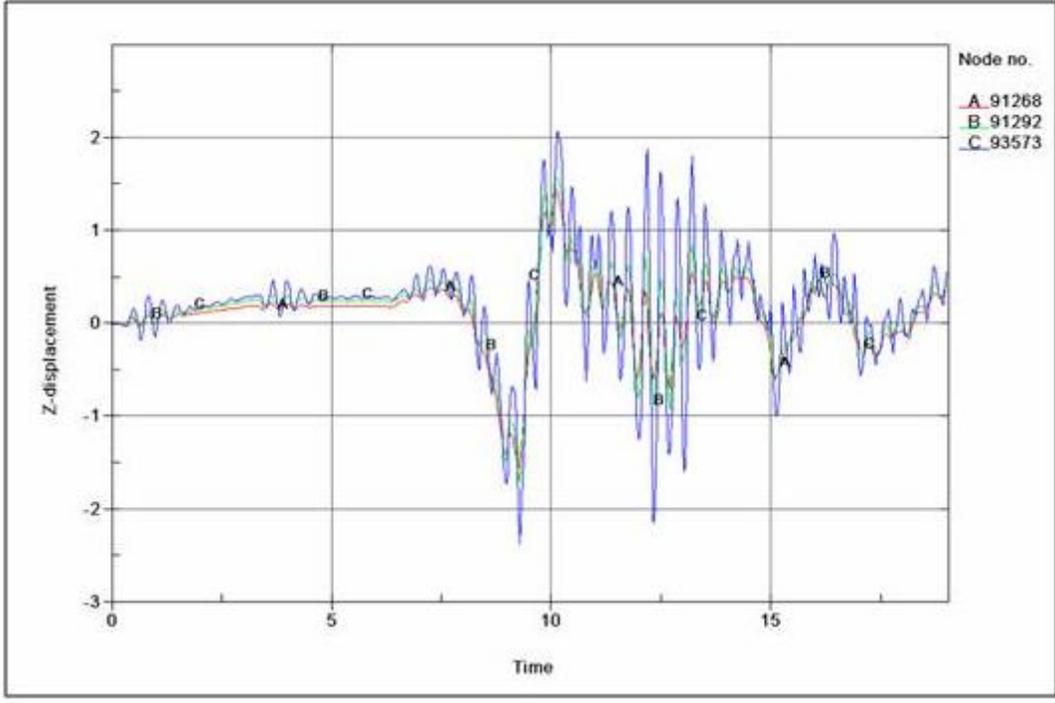
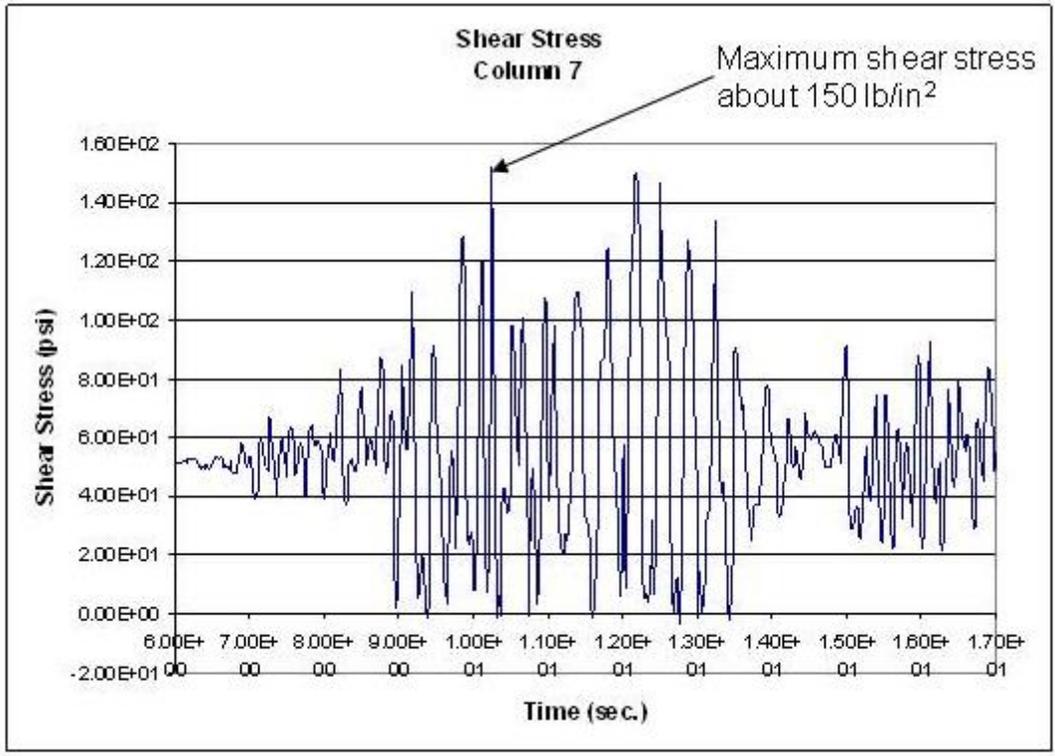
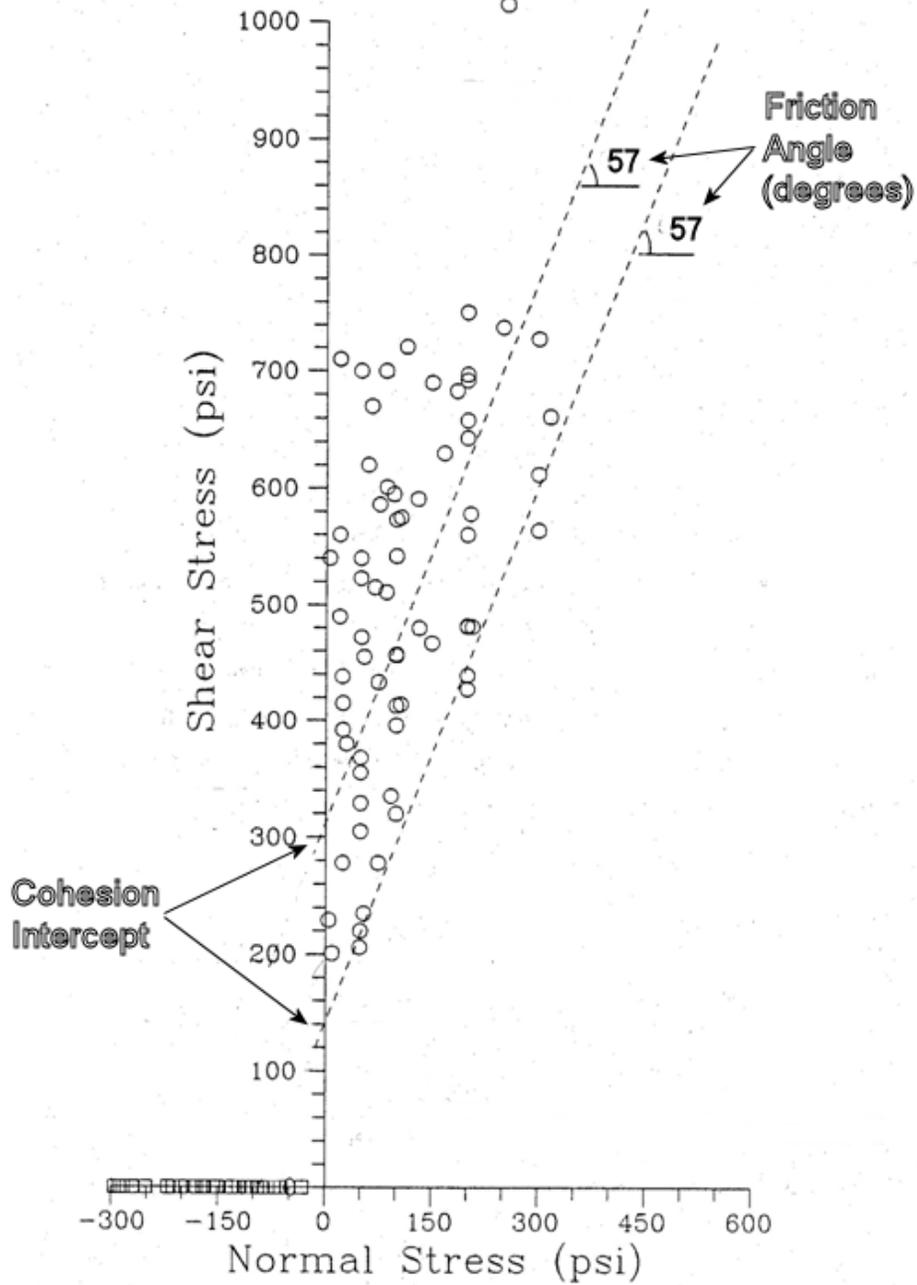


Figure E-2-19 Example Shear Stress and Displacement Time Histories from a Finite Element Analysis.



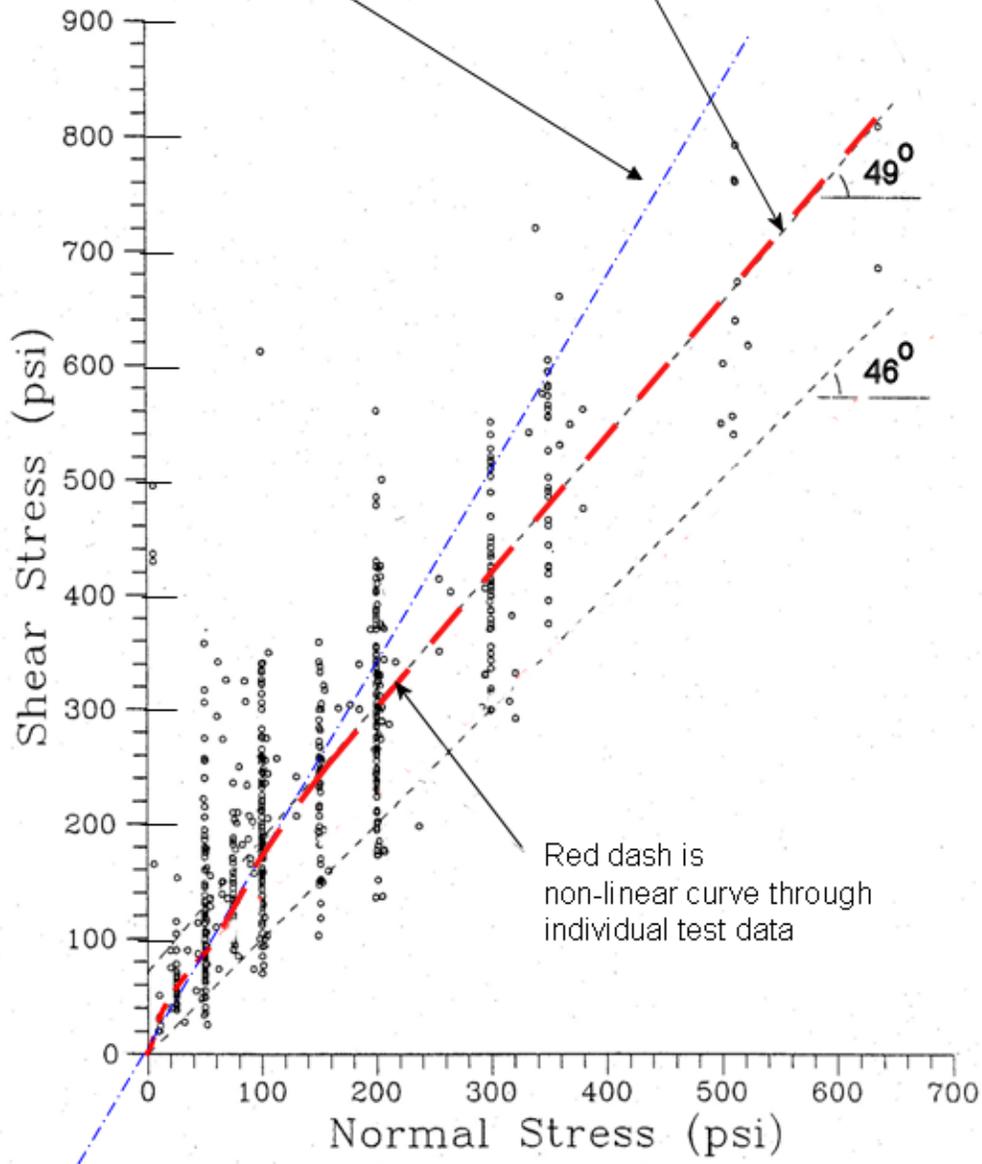
Peak Shear Strength of Concrete-Lift Joints

Adapted from "Uplift Pressures, Shear Strengths, and tensile Strengths for Stability Analysis of Concrete Dams," Volume 1, Electric Power Research Institute, EPRI TR-100345, August, 1992.

Figure E-2-20 Shear Strength for Bonded Construction Joints.

This straight line approximates non-linear curve at low normal stress
 Apparent Cohesion = 0
 Friction = 60 degrees

This straight line approximates non-linear curve at high normal stress
 Apparent Cohesion = 70 lb/in²
 Friction = 49 degrees



Sliding Friction Shear Strength of Concrete-Lift Joints (Unbonded joints)

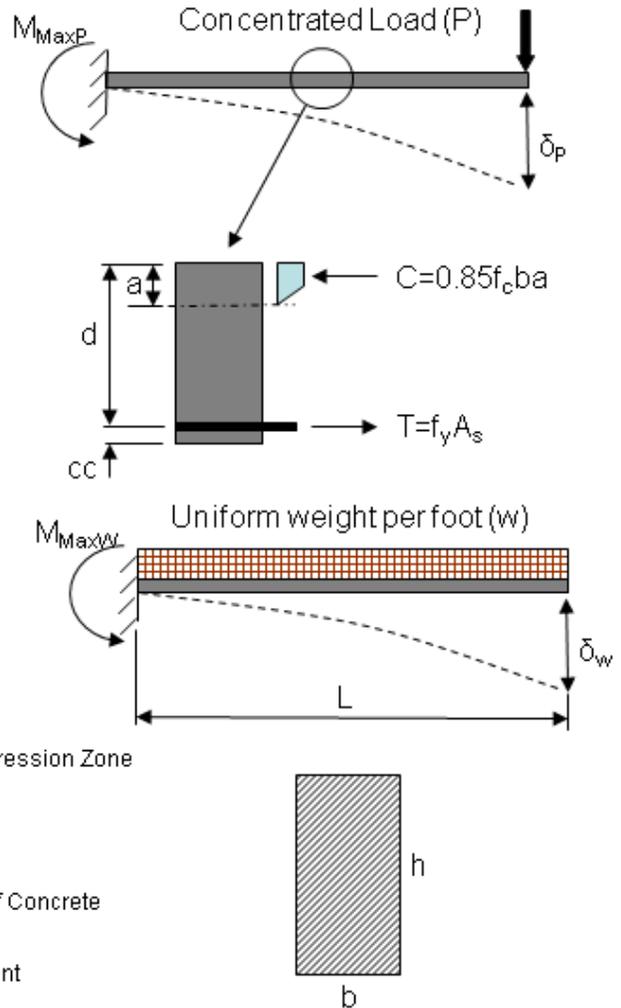
Adapted from "Uplift Pressures, Shear Strengths, and tensile Strengths for Stability Analysis of Concrete Dams," Volume 1, Electric Power Research Institute, EPRI TR-100345, August, 1992.

Figure E-2-21 Shear Strength for Unbonded Construction Joints.

Compute the Maximum Yield Displacement for a Cantilever Beam

Given the Material and Section Properties of the Beam

$b := 1\text{ ft}$	Width of Cantilever
$h := 18\text{ in}$	Depth of Cantilever
$cc := 2\text{ in}$	Clear Cover
$d := h - cc = 16\text{ in}$	Distance to Reinforcement
$L := 20\text{ ft}$	Length of Cantilever
$A_s := 1.56\text{ in}^2$	Area of Steel
$E_s := 29,000,000 \frac{\text{lb}}{\text{in}^2}$	Modulus of Steel
$f_y := 60,000 \frac{\text{lb}}{\text{in}^2}$	Yield Strength of Steel
$E_c := 3,000,000 \frac{\text{lb}}{\text{in}^2}$	Modulus of Concrete
$f_c := 4,000 \frac{\text{lb}}{\text{in}^2}$	Concrete Compressive Strength
$I := \left(\frac{1}{12}\right) \cdot b \cdot h^3$	$I = 5832\text{ in}^4$ Moment of Inertia
$a := \frac{(f_y A_s)}{0.85 f_c b}$	$a = 2.294\text{ in}$ Height of Compression Zone
$T := f_y A_s$	$T = 93600\text{ lb}$ Tension in Steel
$C := 0.85 f_c a b$	$C = 93600\text{ lb}$ Compression of Concrete
$M := f_y A_s \left(d - \frac{a}{2}\right)$	$M = 115853\text{ ft} \cdot \text{lb}$ Balanced Moment



Yield Displacements for a Simply Supported Beam

With a Concentrated Load (P) at Free End

$$M_{MaxP} = PL$$

$$P_P := \frac{M}{L} \quad P_P = 5793\text{ lb}$$

$$\delta_{YieldP} := \frac{(P_P \cdot L^3)}{3 \cdot E_c \cdot I} \quad \delta_{YieldP} = 1.5\text{ in}$$

With a Uniform Load (w) Along Cantilever

$$M_{MaxW} = \frac{wL^2}{2}$$

$$w := \frac{(2 \cdot M)}{L^2} \quad w = 579 \frac{\text{lb}}{\text{ft}}$$

$$\delta_{YieldW} := \frac{(w \cdot L^4)}{8 \cdot E_c \cdot I} \quad \delta_{YieldW} = 1.1\text{ in}$$

Note: Calculations assume a cracked section throughout the length of the beam.

Figure E-2-22 Example Yield Deflection Calculation for a Simple Cantilever Beam.

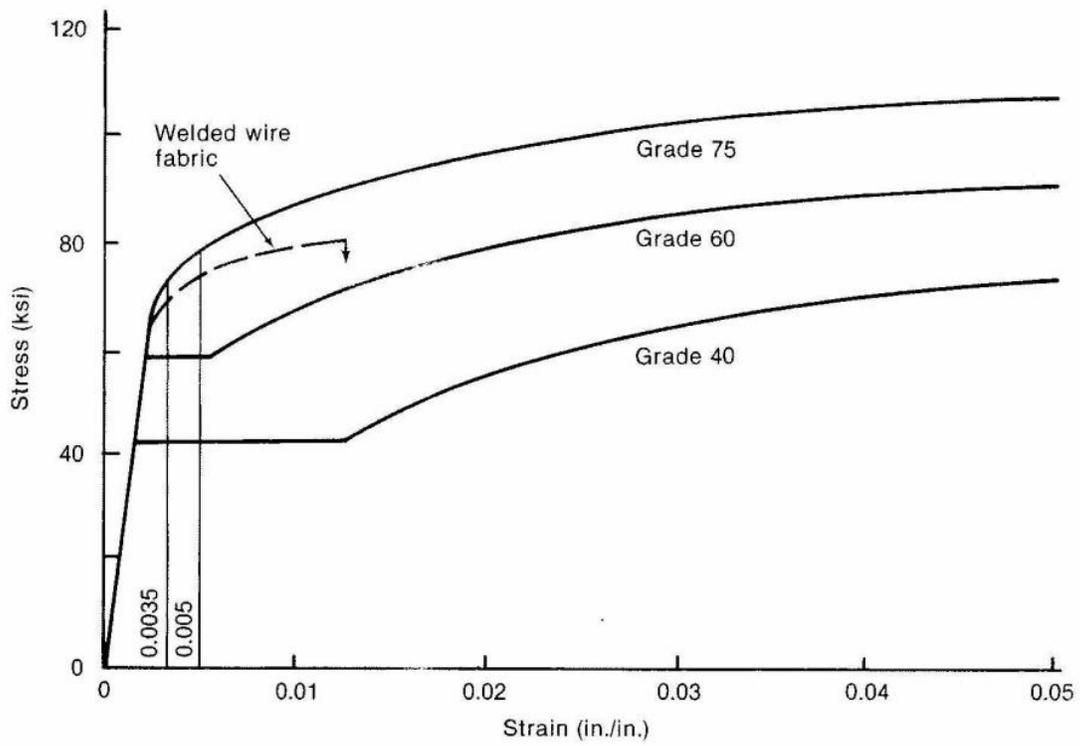
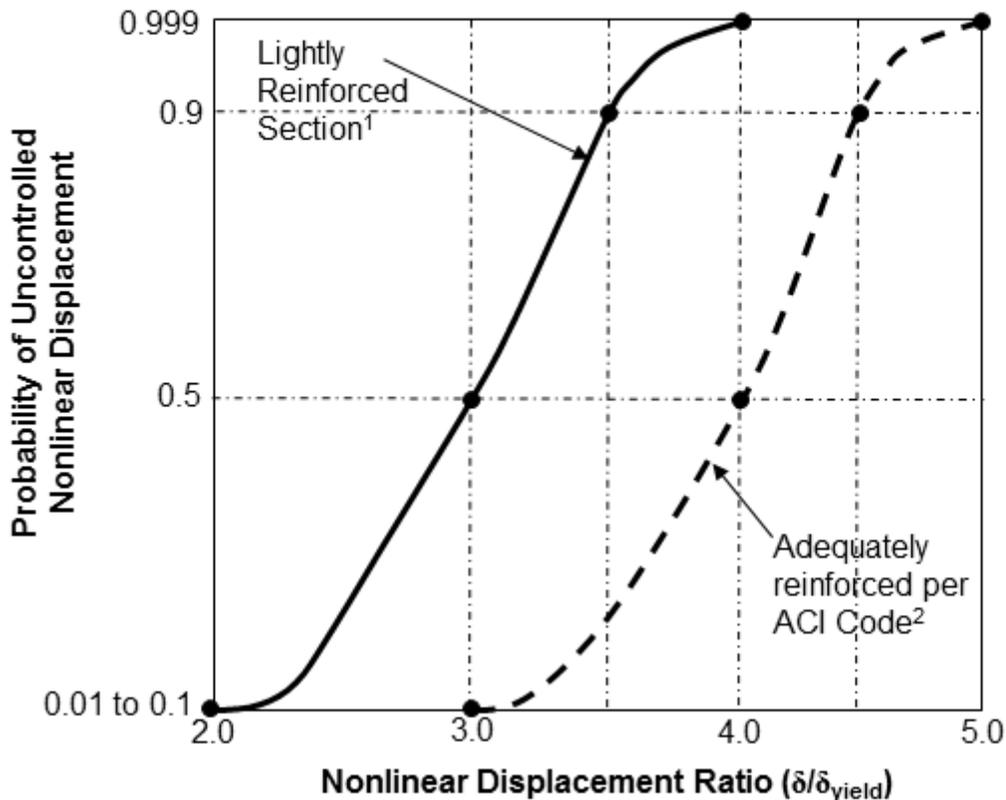


Figure E-2-23 Typical Stress-Strain Curve for Reinforcement.

Uncontrolled Nonlinear Displacement Response



Notes:

1. Lightly reinforced section is defined as a section with $A_s < A_{s(min)}$ in accordance with current ACI 318 code provisions. This represents a condition that could result in a brittle failure when the cracking moment of the unreinforced concrete section computed using its modulus of rupture exceeds the flexural strength of the reinforced concrete section using cracked section analysis.
2. An adequately reinforced section is defined as a section with $A_s \geq A_{s(min)}$ in accordance with current ACI 318 code provisions. In addition, $\rho \leq 0.75\rho_b$ to ensure a ductile flexural failure mechanism is achievable.
3. These nonlinear displacement response curves should only be considered when the section has not failed in shear.
4. The yield displacement (δ_{yield}) is the amount of displacement a member needs to deflect to yield the section reinforcement.
5. Great care shall be taken in considering inelastic displacements of a structure. Specifically, the ability of a concrete structural system to perform well inelastically is highly dependent on member reinforcement details such as embedment length, splice length, hook details and confinement reinforcement.

Figure E-2-24 Nonlinear Displacement Response Curves.

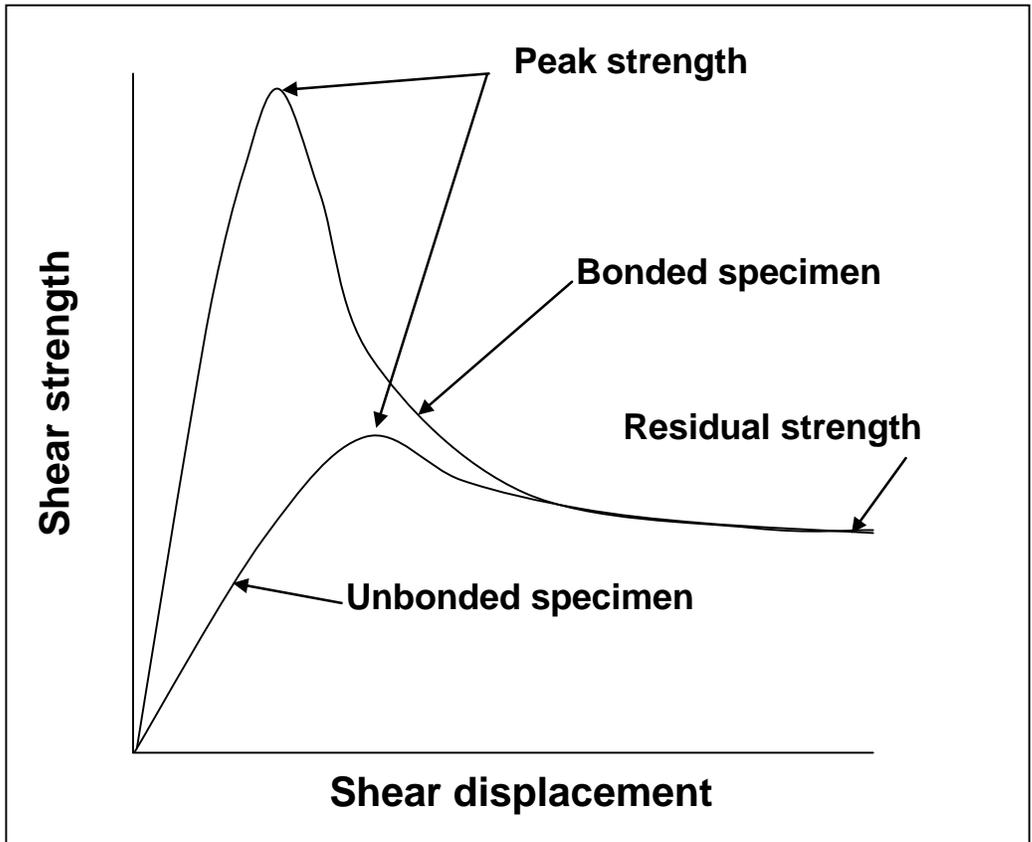


Figure E-2-25 Nonlinear Displacement Response Curve.

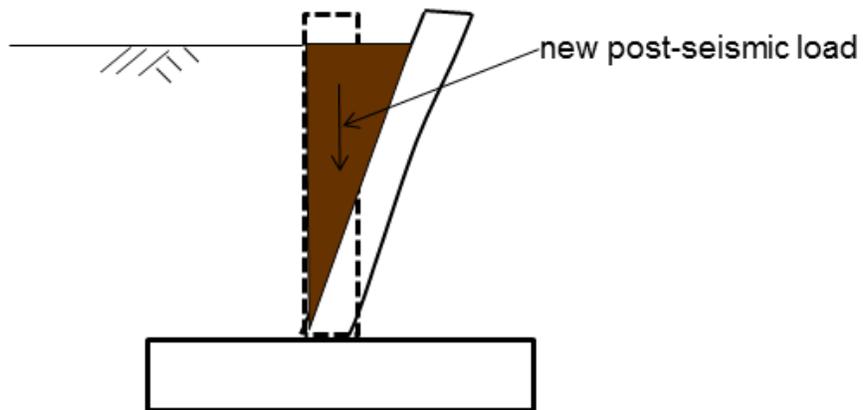


Figure E-2-26 Post-seismic Loading of Yielded Member