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E-3 RISK ANALYSIS FOR CONCRETE GRAVITY STRUCTURES

E-3.1 Key Concepts

Within the context of this chapter, massive concrete spillways or other gravity-type concrete water retention structures are also referred to as concrete gravity dams. Mass concrete dams generate their stability primarily by their inherent weight that works to counteract driving forces. Historically, the leading cause of concrete gravity dam failures (for those founded on rock) has been related to sliding on planes of weakness within the foundation, most typically weak clay or shale layers within sedimentary rock formations. There have also been noted failures along coal seams within the foundation. A few failures have also occurred along weak lift joints within masonry (and buttress) dams. This chapter focuses on risks associated with sliding instability of concrete gravity dams.

For concrete gravity dams founded on alluvial soils, the leading cause of failure is piping or “blowout” of the soil material from beneath the dam. Therefore, the reader is referred to “chapter D-6, Internal Erosion Risks for Embankments and Foundations” for evaluating this potential failure mode, considering “backward erosion piping” of the foundation soils.

The heel of the dam is a location of sharp geometry change and as such is a point of singularity and stress concentration. Thus, the dam-foundation contact is typically the focus of most of the stability analyses. However, this typically is not the weak link in the dam-foundation system, unless the dam is founded on a foot wall of smooth discontinuity surfaces such as faults or bedding planes. The rough surface that results from blasting the dam keyway excavation typically provides a significant roughness or “dilation” component to the shear strength on this surface, which should be taken into account to the extent possible based on construction photographs and other information. If the surface clean-up is good, significant cohesion and tensile strength can result (as with lift joints).

When surface cleanup of lift joints is not good, weaker horizontal planes may occur within the dam body. For gravity dams constructed in blocks, the weaker planes may not “line up” across contraction joints, and if the joints are constructed with keys, considerable stability can result from load transfer to adjacent monoliths. This should be considered when evaluating the risks associated with monolith instability.

A line of functioning drainage holes in the foundation or dam body adds significantly to the sliding stability of concrete gravity dams by reducing water pressures (typically referred to as “uplift”) along potential sliding surfaces. A

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decrease in water pressures increases the effective normal stress and frictional resistance. Research shows that drains can remain effective even if a crack or open surface extends downstream of the drainage curtain as noted in nonlinear analysis guidelines (Koltuniuk et al. 2013), based on the Electric Power Research Institute research results (Amadei et al. 1991). However, drainage systems can become plugged over time if they are not maintained, and the drainage curtain can be offset under significant seismic displacements, thus reducing the drain effectiveness.

Shear keys constructed within the contraction joints separating concrete monoliths are beneficial in that they can facilitate load transfer between monoliths. This could be important if one monolith or series of monoliths contains an unbonded lift joint or weak foundation conditions, whereby load in excess of the weak monolith(s) capacity could be transferred to adjacent stronger monoliths. Not all gravity dams contain shear keys within the contraction joints, but some do, and this should be considered as part of the overall evaluation.

When a potential sliding plane is formed by a partially bonded and partially unbonded surface, care must be taken in assigning the shear strength to each portion. That is because the peak shear strengths may not be mobilized at compatible displacements. It may take much less shear displacement to mobilize the shear strength of a bonded joint than an unbonded joint, in which case it may not be appropriate to simply add the peak strengths determined from testing. Test results could be examined and new strength curves developed at compatible displacements.

A special discussion on the so called “cracked base” analysis is provided here, as estimating risks requires a somewhat different approach than that currently provided in design criteria documents. As opposed to designing a new dam, where conservative assumptions and criteria are appropriate to ensure that the dam does not slide for the design loads, estimating risks for an existing dam requires attempting to establish the most likely behavior and include variability of important parameters to account for uncertainty. In order to estimate the cracking behavior of the dam the concrete tensile strength must be established. For a detailed discussion of concrete tensile strength refer to “chapter E-1, Concrete Property Considerations.”

E-3.1.1 Cracked Base Analysis

The “cracked base” analysis has found its way into most concrete gravity dam design criteria, based on the “gravity” method of analysis, which assumes plane sections remain plane, and thus the distribution of vertical stress is linear. It is often applied without thoroughly evaluating the reasonableness of the results or the analysis assumptions relative to actual conditions. In a risk context, these must be considered. Several important points in this regard include:

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- There is often confusion in how to deal with total stress and effective stress in carrying out the calculations. *Design of Small Dams* (Bureau of Reclamation 1987) indicates that “Uplift from internal water pressures and stresses caused by the moment contribution from uplift along a horizontal plane are usually not included in the computation of σ_z .” This is the total stress method, which is endorsed by Watermeyer (2006), who states that the “reactive stress equations [which include the contribution from uplift] are erroneous and can lead to erroneous conclusions when uplift reducing drains are incorporated into the base of a gravity dam.” That is not to say that uplift is not considered in the analysis, only that the moment contribution from internal uplift forces are not included in the stress calculations.
- The effective stress is determined by subtracting the pore water pressure (often equated to the “uplift pressure”) from the total stress. If the effective stress is tensile and exceeds the tensile strength, then it is assumed that cracking can initiate. At that point, the water force in the crack becomes an “external” force which is included in the *total stress* calculations, and the base length is assumed to be shortened to only that portion downstream of the crack tip. The effective stress at the crack tip is subsequently calculated as the difference between the total stress and effective stress at that location. ***It should be noted that the crack may not progress downstream of the point at which the effective stress is equal to the tensile strength.***
- At the base of the dam, the potential for full reservoir pressure at the crack tip is controlled by the permeability of the foundation. Concrete gravity dams are typically founded on fractured and jointed rock. Thus, full reservoir pressure cannot develop at the tip of a crack along the foundation contact unless the foundation rock is massive and un-fractured, or the foundation joints are much tighter than the base crack. This is because water entering the crack will flow out through fractures at the base of the dam, and head loss will occur due to this flow. Thus, full uplift in a crack tip at the foundation contact may not be reasonable.
- Drains remain effective even if penetrated by a horizontal crack, although the drain efficiency may be reduced somewhat. This is demonstrated by the research sponsored by the Electric Power Research Institute at the University of Colorado (Amadei et al. 1991). Thus, analyses which consider full hydrostatic reservoir pressure in a crack tip downstream of the line of drains are typically not used for risk analyses until displacements greater than the drain diameter have occurred and there is an assumption that the drains are no longer effective. Even when drains have displaced greatly, they can still be effective to reduce some water pressures.

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- In the limiting case, if a crack is judged to propagate completely through the structure, the uplift pressure distribution along the crack is that which is appropriate for the post-cracking conditions, including the effects of drains in reducing the pressures, and pressures no higher than tailwater at the downstream face. It should be noted that there is very little guidance currently available concerning the effects of drains if the section cracks all the way through. If the aperture of the crack is thought to remain relatively tight in comparison to the drain diameter, the drains should retain some effectiveness. If the aperture is thought to be large in comparison to the drain diameter, then there may be more flow than the drains can handle, and their effectiveness would be questionable.
- If a crack is shown to exist, cohesion is presumed to act only on the portion of the intact potential sliding plane that is in compression. It is expected that intact concrete in tension will exhibit a smaller cohesive strength component, and since this is difficult to quantify, it is typically ignored.

E-3.2 Risks Under Normal Operations

Concrete gravity dams that have performed well under normal operating conditions will likely continue to do so unless something changes. Changes could result from plugging of drains leading to an increase in uplift pressures, possible gradual creep that reduces the shear strength on potential sliding surfaces, or degradation of the concrete from alkali-aggregate reaction, freeze-thaw, or sulfate attack. These may be difficult to detect. A review of instrumentation results can be helpful. For example, if piezometers or uplift pressure gauges indicate a rise in pressures, and weirs indicate a reduction in drain flows, the drains may be plugging leading to potentially unstable conditions. If conditions appear to be changing, risk estimates are typically made for projected conditions as well as current conditions. The projected conditions should be evaluated within the context of the evaluation as some studies required different study periods for the risk analysis. Refer to the agency specific evaluation for criteria.

Reliability analysis for sliding on near horizontal foundation planes and/or potentially weak or cracked horizontal lift joints, typically using two-dimensional analysis sections, is the primary tool used for estimating risks posed by concrete gravity dams under normal operating conditions. This involves performing a probabilistic stability analysis using the Monte-Carlo technique as described in “chapter A-7, Probabilistic Approaches to Limit-State Analyses” It requires an assessment of the likely range in input parameters, such as drain efficiency, cohesion and friction coefficient along the potential sliding surface, percentage of potential sliding surface that is intact, orientation of the potential sliding surface, and unit weight of the material(s). For potential foundation sliding planes, the

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influence of a downstream passive rock wedge should be considered, where appropriate. This will depend upon the depth of the monolith embedment, quality of the rock, etc. The shear strength of rough surfaces is nonlinear as a result of “riding up” over asperities at low normal stress and shearing through them at high normal stress. A straight line fit through such data points can result in overestimating the shear strength, particularly at low normal stresses. Therefore, strength parameters should be selected for the appropriate normal stress range of interest, or other means used to account for the nonlinear shear strength envelope.

Probabilistic stability analyses are typically performed at various reservoir water surface elevations, and combined in an event tree, such as that shown on figure E-3-1 (see “chapter A-5, Event Trees” and “chapter B-1, Hydrologic Hazard Analysis” for information on calculating reservoir load range probabilities). Note that in the limit, if small enough reservoir elevation increments are selected, a curve, referred to as a “fragility curve”, results. The calculations are essentially the same whether larger discrete ranges or a fragility curve is used, and the results are similar as long as care is taken in selecting the discrete ranges. Therefore, either method can be used in estimating risks.

For the probabilistic stability analyses, it is important to examine the sensitivity of the coefficients varied in analysis and perform parametric studies, varying the parameters that affect the results the most. These parametric studies are used to estimate an appropriate range in conditional failure probabilities for the node titled “Sliding Instability”. If there are significant three-dimensional effects, the two-dimensional sliding model may not be appropriate, and three-dimensional analyses may be needed to get a handle on how significant these effects might be if risks estimated from the two-dimensional models exceed the public protection guidelines.

E-3.3 Risks Under Flood Loading

The approach for estimating risks due to structural instability under flood loading is essentially the same as for static loading, except that reservoir water surface elevations above the normal operating range, assigned the appropriate flood frequency, are used in the analyses and event tree. If flood routing information is not available, a conservative initial assumption is that inflow is equal to outflow, and the level of the reservoir is determined by that needed to pass a given peak inflow through the spillway and/or other release facilities (see also “chapter D-3, Flood Overtopping Failure of Dams and Levees”). If the risks using this method are in an area where risk reduction actions are justified, flood routings may be needed to get a better handle on the probability of attaining various reservoir elevations.

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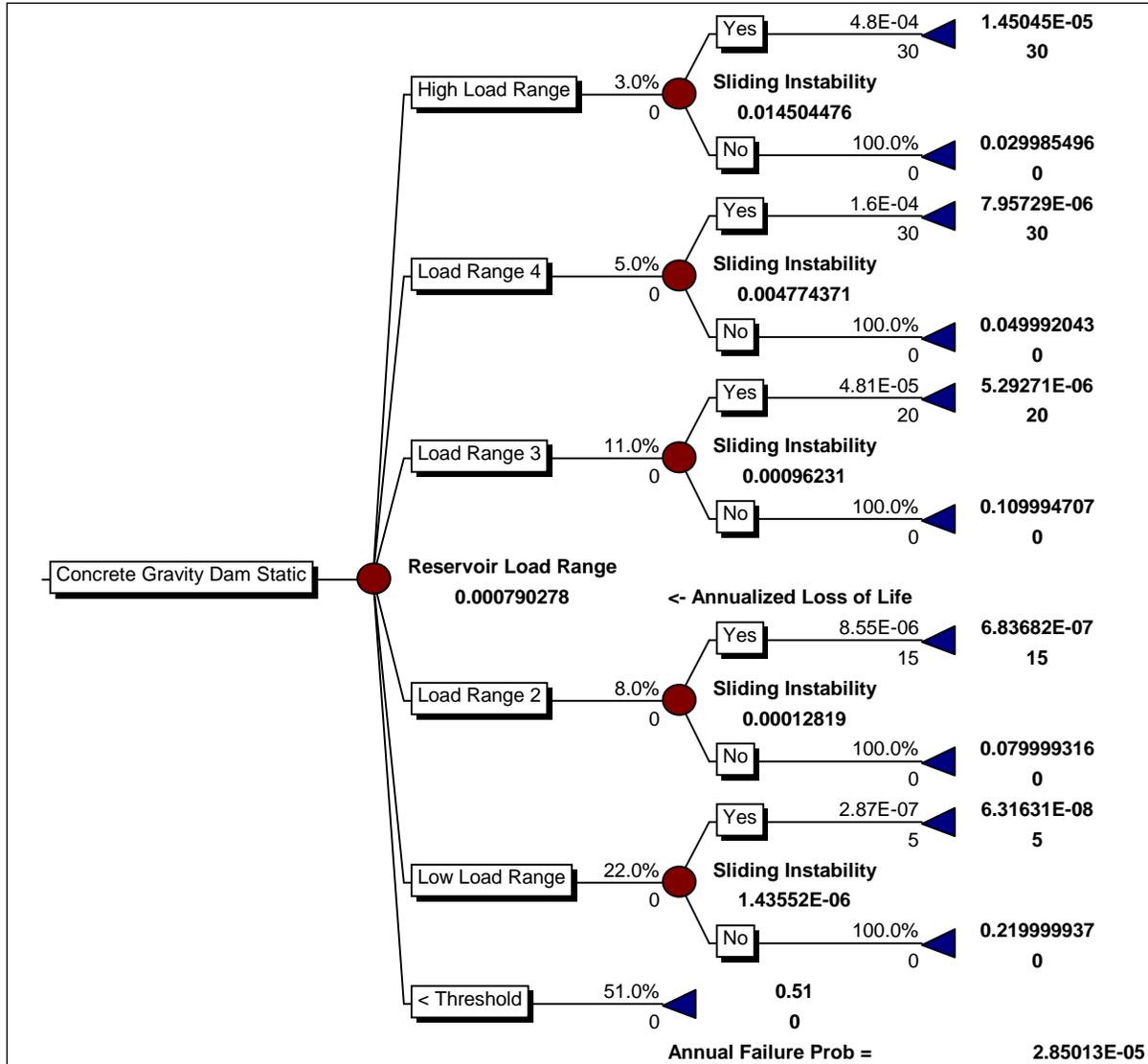


Figure E-3-1.—Concrete gravity dam instability, static load event tree.

As the reservoir rises during flood loading, there may be a level at which the heel of the dam goes into tension (based on effective stress), in which case the potential for cracking along a lift joint at that elevation may increase. At some point, the estimated tensile strength of the concrete may be exceeded. Typically, a separation in the event tree reservoir load ranges occurs at these reservoir elevations. Stability analyses should be performed at these reservoir water elevations to judge the impact on the dam. Make sure the tailwater and uplift conditions correspond to the given reservoir elevation. In the case of an overflow section, care must be taken when assuming nappe forces (forces due to water flowing above the spillway) and tailwater forces act on the dam. Stilling basins can “sweep out” at high flows, and nappe pressures can become subatmospheric,

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reducing the stabilizing forces. Forces generated by water flowing through a flip bucket can also affect the results. A hydraulic evaluation is typically performed to determine whether it is appropriate to include these forces. A reliability model with the proper formulation for a cracked base analysis (see Watermeyer 2006) is important in examining conditions where tension exceeding the tensile strength develops.

Risk of foundation block failure in the foundation of a gravity dam should also be considered for flood conditions. If a potentially removable block with weak planes is present under a gravity dam, a failure similar to Bayless Dam, PA could present considerable risk to the structure. Increases in uplift due to flooding and increased reservoir elevations should be incorporated in analyses for stability of the section. For more information on this potential failure mode, see “chapter E-4, Risk Analysis for Concrete Arch Dams.

Risk evaluation associated with overtopping erosion of the abutments or foundation is discussed in “chapter D-3, Flood Overtopping Failure of Dams and Levees” and “chapter D-1, Erosion of Rock and Soil.” However, another potentially significant issue involves cases where a concrete gravity dam serves as a spillway section. If erosion occurs at the downstream toe of the structure during spillway releases, weak bedding planes or foundation discontinuities in the underlying foundation rock might be exposed, daylighting into the erosion hole. This could remove passive resistance from the downstream rock mass and result in a much more unstable condition. See “chapter D-1, Erosion of Rock and Soil” for guidance on how to estimate the potential for erosion. Figure E-3-2 shows how this might impact the event tree. The post erosion/scour stability analysis would follow the same general procedures as outline previously within this chapter but would have to reflect the change in the foundation characteristics. The potential for failure of stilling basins is discussed in “chapter F-2, Overtopping of Walls and Stilling Basin Failure.”

E-3.4 Risks Under Earthquake Loading

Under earthquake loading, concrete gravity dams will respond according to the level and frequency of the shaking, and the reservoir level at the time of shaking. Therefore, sufficient analyses need to be performed to evaluate conditional failure probabilities at various levels of shaking and reservoir elevation. An example event tree to examine the potential for sliding failure through a weak lift line at a sharp change in slope on the downstream face is shown on figure E-3-3.

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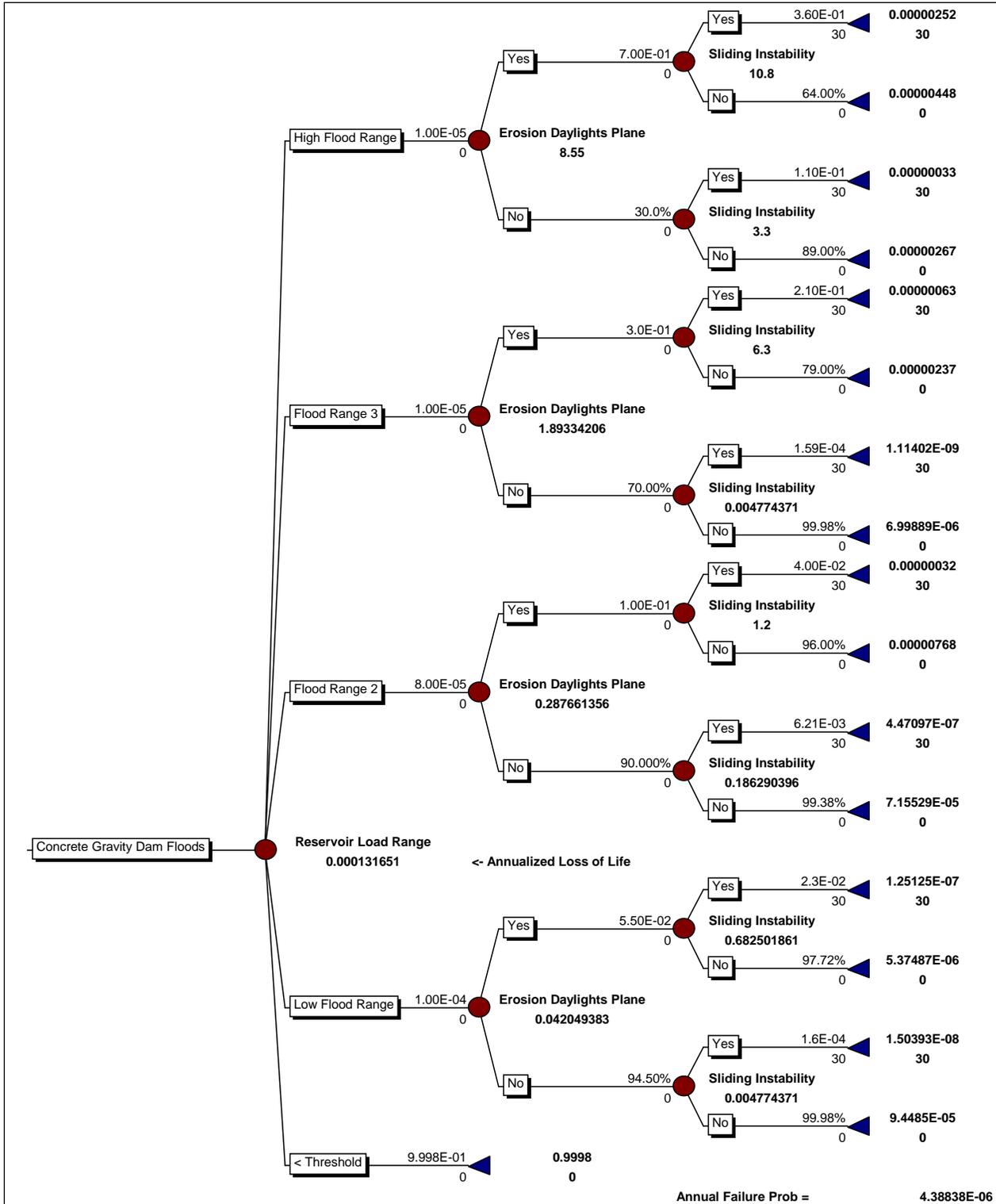


Figure E-3-2.—Concrete gravity dam instability, flood loading event tree.

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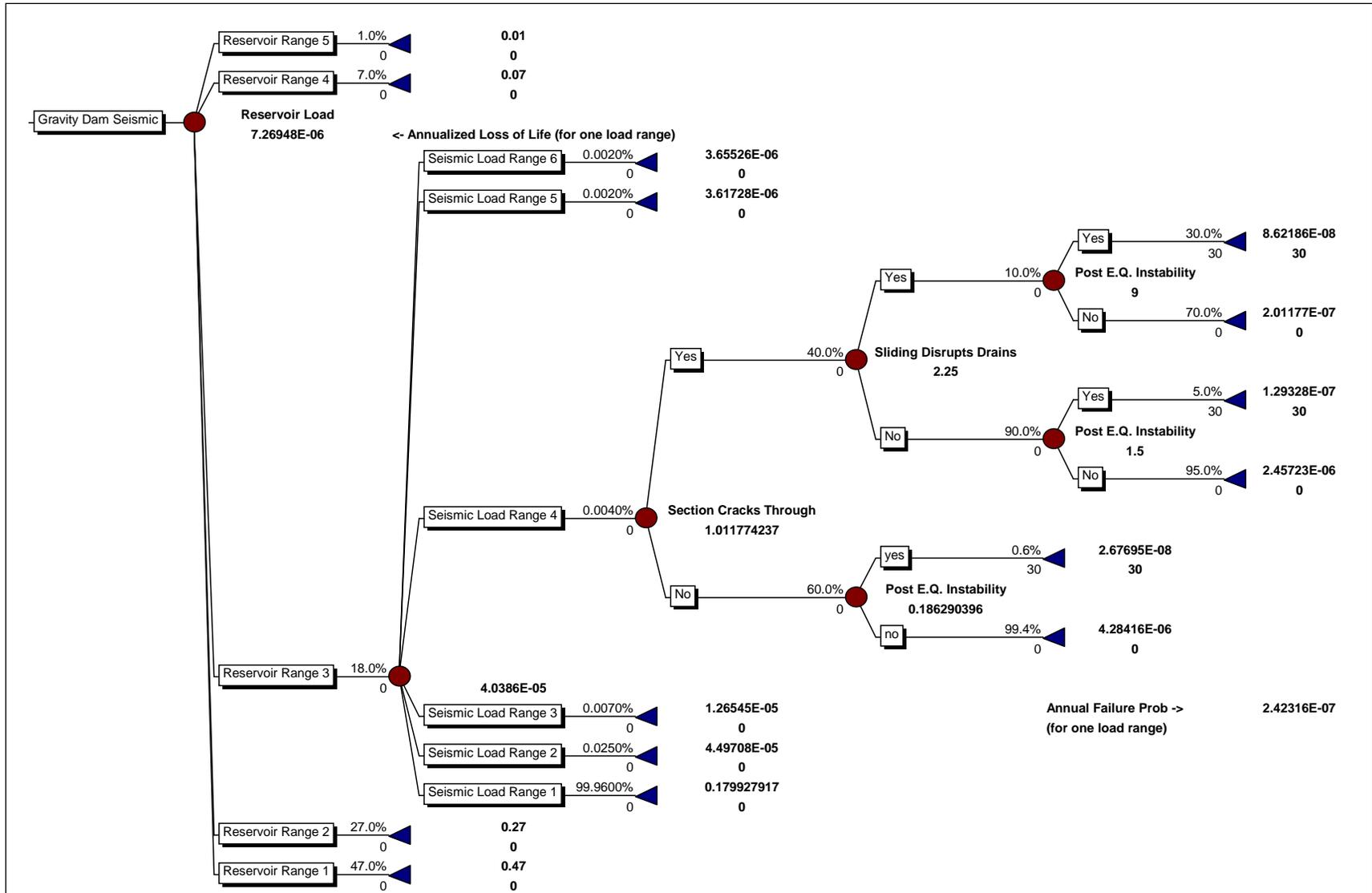


Figure E-3-3.—Concrete gravity dam instability, seismic loading.

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For each reservoir and seismic load range that is established for the estimating process, the likelihood of cracking through the dam body at this location must be estimated. The best approach for this is to perform a nonlinear dynamic finite element studies, modeling the potential weak plane with a contact surface that can be assigned a tensile strength value. As the tensile strength is exceeded near the faces during seismic response, the nodes will separate. If the shaking is severe enough, complete separation of the contact surface may propagate through the structure. Figure E-3-4 shows a horizontal contact surface through a three-dimensional model of a concrete gravity dam. The darker color represents portions that remained uncracked following the earthquake shaking. This indicates that at least one monolith cracked completely through for the set of assumptions used in this analysis. Similar studies can be performed using a two-dimensional section. By varying the tensile strength within reasonable parameters and monitoring the percentage of the joint that separates, a range in the likelihood of complete separation can be made. It should be noted that this is a total stress analysis, and pore pressures are not considered. Pore pressure behavior in concrete under dynamic loading is a subject of much uncertainty. Therefore, it is typically assumed that the total stress analysis provides a reasonable approximation of the potential for cracking through the section.

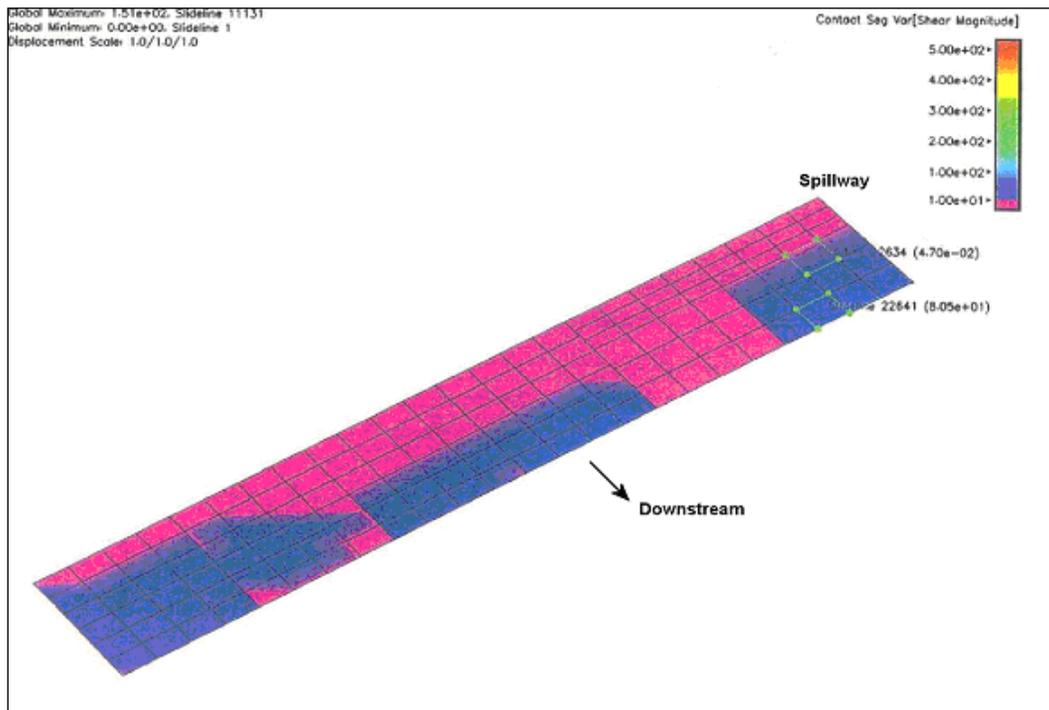


Figure E-3-4.—Separation of contact surface in dynamic finite element analysis (lighter color indicates separation).

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If the dam only cracks partially through, the probability of post-earthquake instability in the estimated cracked state is determined using static reliability analysis, as previously described. The estimated crack length from the nonlinear analysis of the seismic shaking is used as the starting point for a cracked base analysis. It is very difficult to estimate the amount and depth of cracking from a linear analysis. Linear analyses only help determine if and where cracks might initiate (high stress areas) but cannot model crack development or the sudden release of kinetic energy when cracks form. If there is significant uncertainty introduced to the risk estimates by using only linear analyses, non-linear analyses can be performed to examine the effects of non-linear material models in the progression of cracking through the section. This type of analysis is complex and costly and should be initiated only when needed to better understand significant uncertainty associated with high risk estimates.

If the section cracks all the way through, the likelihood of shearing the drains is next estimated. Information typically used to make this assessment includes calculated displacements from the finite element study assuming frictional resistance only on the potential sliding surface, as shown on figure E-3-5. In this case, very small values of damping, only enough to keep the model stable as the loading is applied, need to be used. If the model is over-dampened, the displacements will be under-estimated. Although this type of analysis assumes the section is cracked at the beginning of the earthquake and thus are somewhat conservative, they can be used to estimate the likelihood of drains, where present, being sheared. The post-earthquake instability could be considerably different whether the drains are still functioning after the earthquake shaking or not. It is possible that the drains could be sheared off, or opening of pathways in the foundation could lead to increased flow that overwhelms the drainage system. Therefore, two estimates are made, using reliability analysis, to account for these two conditions (drains functional or not), as indicated by the nodes on the event tree on figure E-3-3.

Seismic risk analysis of concrete gravity dams typically relies heavily on finite element analyses to evaluate the dynamic response, and the “gravity method” analyses to evaluate post-earthquake stability. The finite element analyses described above are not routinely performed. Although more uncertain, if analyses that include a contact surface are not available, it may be necessary to make judgments on cracking from traditional linear elastic finite element analysis results, by examining the magnitude and duration of the vertical tensile stresses at the upstream and downstream faces. Judgments must be made concerning how load is redistributed if cracking begins at the face, and how far toward the center of the dam it will progress, which is not an easy task. It is also important to examine the three-dimensional effects and, for example, whether excess driving load can be transferred to adjacent monoliths through shear keys. This is particularly true if all analyses are based on two-dimensional sections.

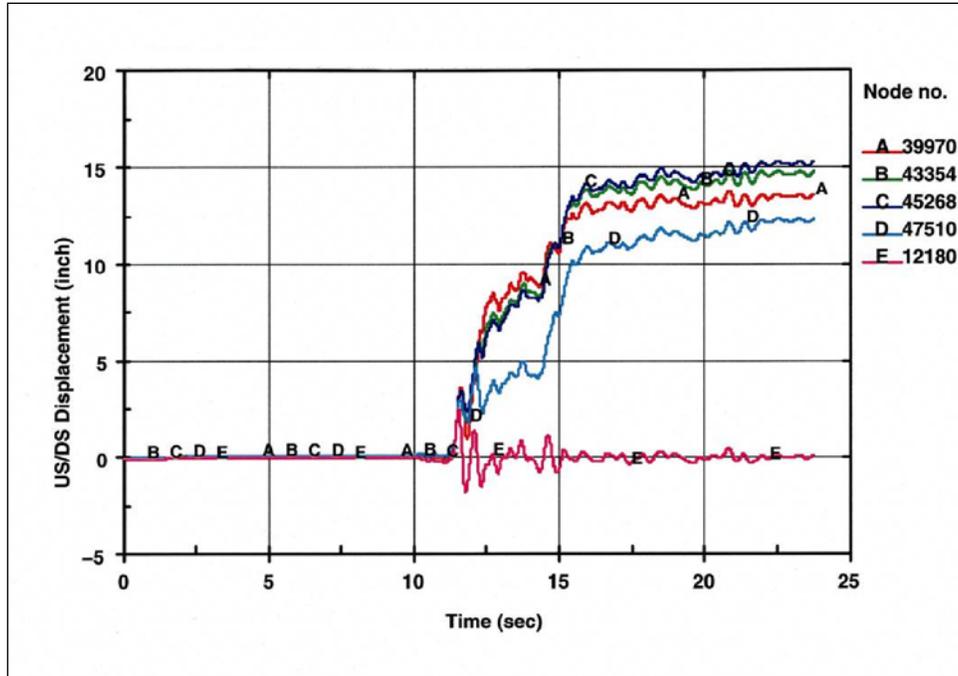


Figure E-3-5.—Displacement of various monoliths during dynamic loading (friction only, curve e is at base of sliding contact surface).

Lacking dynamic sliding analyses, a first approximation to the magnitude of displacement can be obtained from the following equations (Hendron et al. 1980).

$$\delta = \frac{6V^2}{2gN} \quad \text{for } N/A < 0.2 \quad \text{Equation E-3-1}$$

$$\delta = \left(\frac{V^2}{2gN} \right) \left(\frac{A}{N} \right) \quad \text{for } 0.2 < N/A < 0.4 \quad \text{Equation E-3-2}$$

$$\delta = \left(\frac{V^2}{2gN} \right) \left(1 - \frac{N}{A} \right) \left(\frac{A}{N} \right) \quad \text{for } N/A > 0.4 \quad \text{Equation E-3-3}$$

where g is the acceleration due to gravity, A is the peak ground acceleration as a fraction of gravity, V is the peak ground velocity, and N is the yield acceleration coefficient (expressed as a fraction of gravity and determined from a “gravity analysis” as the seismic coefficient that results in a factor of safety equal to 1.0 with all static loads applied to the structure). These equations are thought to be conservative in most cases. They were developed by Professor Newmark to delineate the upper bound of displacements for slopes from a large range in ground motions, consisting of soil records with significant low frequency content. Thus, longer pulses exceeding the yield acceleration were incorporated into their

development than would be expected for rock records associated with gravity dams. However, the equations were developed from rigid-plastic analyses, and if there is significant structural response associated with a dam with respect to the applied ground motions, the displacements could possibly be larger.

E-3.5 Accounting for Uncertainty

Uncertainty is accounted for by estimating a range or distribution of values for each node on the event tree. A Monte-Carlo analysis is then run for the event tree to display the “cloud” of uncertainty, as described in “chapter A-8, Combining and Portraying Risks.” It is important to perform parametric or sensitivity analyses to examine how the results might change with different input parameters, especially for reliability analyses as described in “chapter A-7, Probabilistic Approaches to Limit- State Analyses.” Different assumptions on the distribution and magnitude of water forces following an earthquake are typically made, since there is typically a great deal of uncertainty surrounding these values, and they can have a controlling effect on the results of the analyses. The uncertainty associated with how well the models are thought to actually predict the complex behavior should also be factored into the estimates, perhaps in a parametric sense (i.e. vary the corrections to account for model uncertainty and examine the results on the risk estimates).

E-3.6 Relevant Case Histories

E-3.6.1 Austin (Bayless) Dam: 1911

Austin Dam was a concrete gravity dam about 43 feet high and 534 feet long constructed by the Bayless Pulp and Paper Company about 1½ miles upstream of the town of Austin, Pennsylvania. A four-foot-thick by four-foot-deep concrete shear key was constructed into the horizontally bedded sandstone with interbedded weak shale layers. Anchor bars were grouted 5 to 8 feet into the foundation, extending well up into the dam body, on 2-foot 8-inch centers, located at about 6 feet from the upstream face. No drains were provided for the dam or foundation. During initial reservoir filling in 1910, the center portion of the dam at the overflow spillway section slid downstream about 18 inches at the base and 31 inches at the crest. The reservoir was lowered, but no repairs were made, and the dam was put back into service. As the reservoir filled again, the dam suddenly gave way on September 30, 1911. More than 75 people lost their lives in Austin. Back analysis suggests that sliding occurred on a weak shale layer within the foundation (Anderson et al. 1998).

E-3.6.2 Bouzey Dam: 1895

Bouzey Dam was a 72-foot high masonry gravity dam constructed across the L'Aviere River near Epinal, France. Similar to Austin Dam, the dam was founded on horizontally interbedded sandstone and lenticular clay seams, with no drainage provisions, and about a 6-foot wide by 10-foot deep cutoff key constructed into the rock at the upstream face of the dam. Also similar to Austin Dam, an incident occurred during initial filling whereby the center section of the dam moved downstream about a foot, shearing the key. Unlike Austin Dam, the reservoir was lowered, and the lower portion of the dam was strengthened. Unfortunately, the upper portion of the dam was quite thin (less than 18 feet thick for about the upper 35 feet), and upon refilling, the dam cracked and the upper 30 feet or so was sheared off and swept away. Stability calculations indicate that cracking was likely at the elevation where the shear failure occurred, and once cracked through, the upper portion of the dam was unstable (Anderson et al. 1998).

E-3.6.3 Koyna Dam: 1967

Koyna Dam is a 338-foot-high and 2,800-foot long concrete gravity dam constructed on the Koyna River in southwestern India between 1954 and 1963. During construction the decision was made to raise the dam and the downstream slope of the non-overflow section was steepened in the upper 120 feet of the structure to accommodate the raise, resulting in a discontinuous change in slope at that location. The dam was shaken by a M6.5 earthquake on December 11, 1967. A strong motion accelerograph located in a gallery on the upper right abutment recorded a peak ground acceleration of 0.63g cross-canyon, 0.49g downstream, and 0.34g vertical. Although the dam did not fail, deep horizontal cracks formed throughout the upstream and downstream faces near the change in slope where a stress concentration is expected to occur, requiring the installation of tendons and construction of buttresses on the downstream face to stabilize the structure. Finite element analyses indicated stress concentrations near the change in slope that exceed the dynamic tensile strength of the concrete (Anderson et al. 1998).

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